Linear Likelihood-based Imprecise Regression (LIR) with interval data

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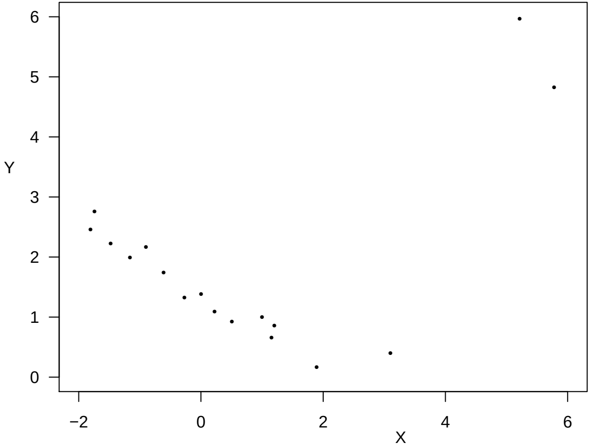
September 11, 2012

(Simple) linear LIR with precise data



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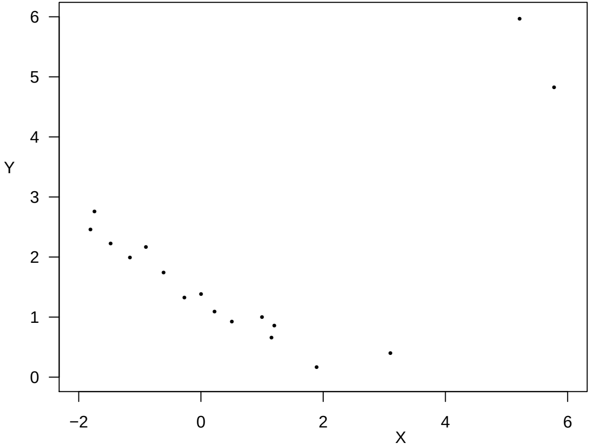

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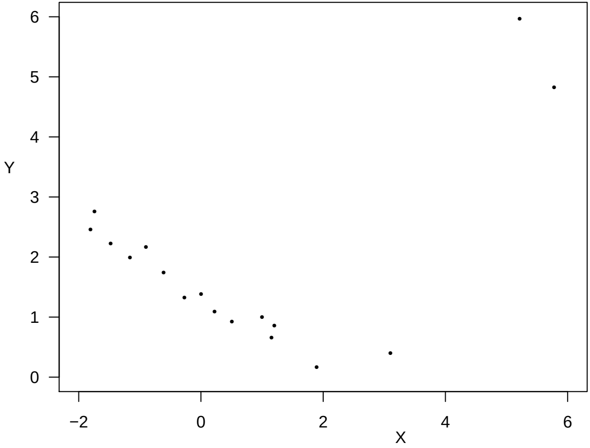

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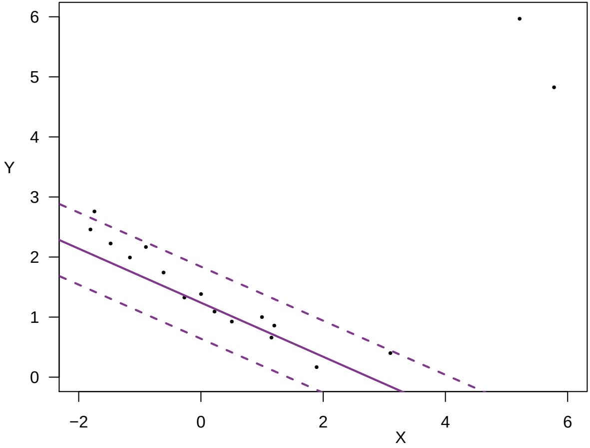

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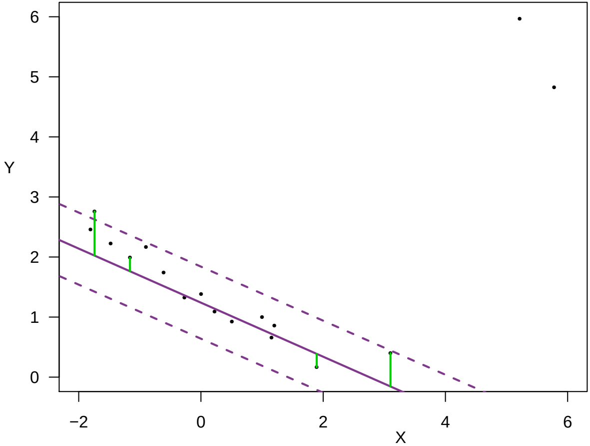

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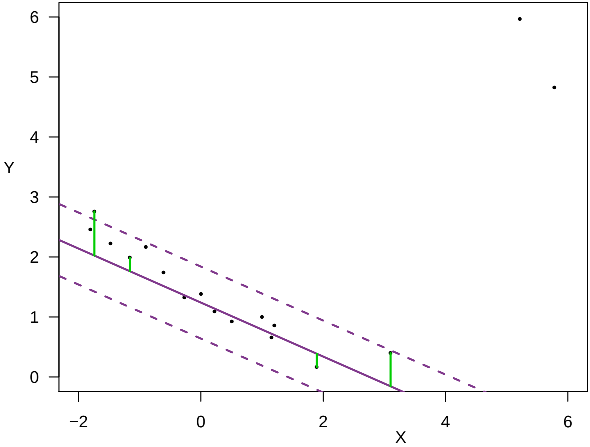

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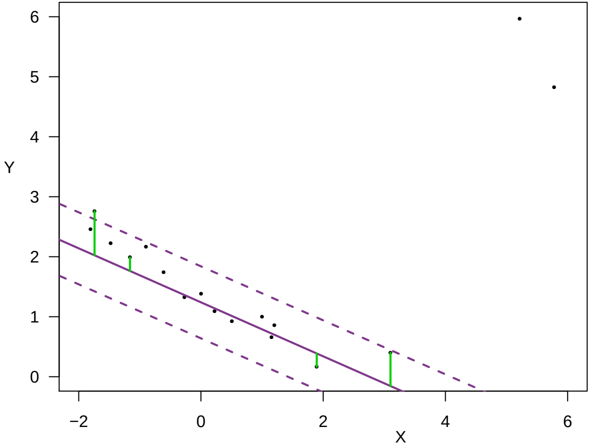

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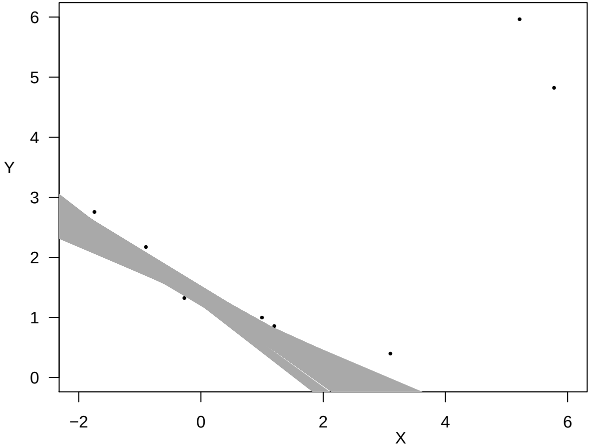

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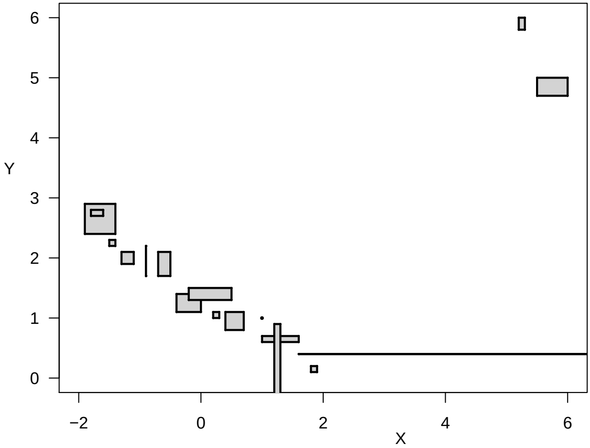

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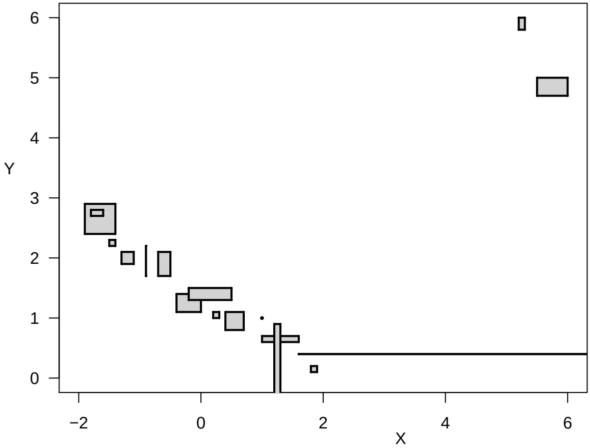

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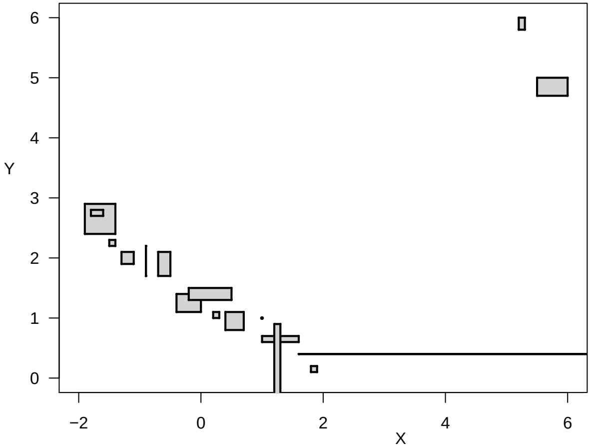

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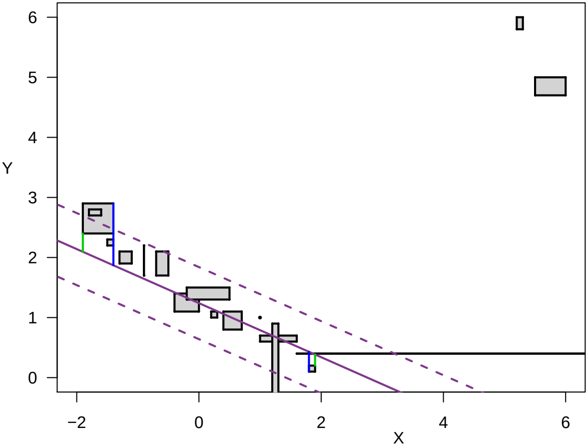

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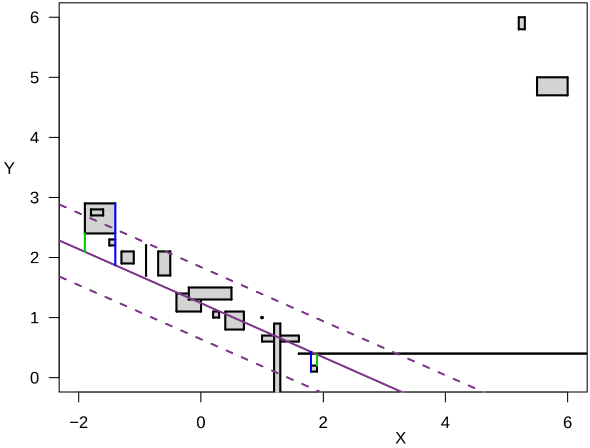

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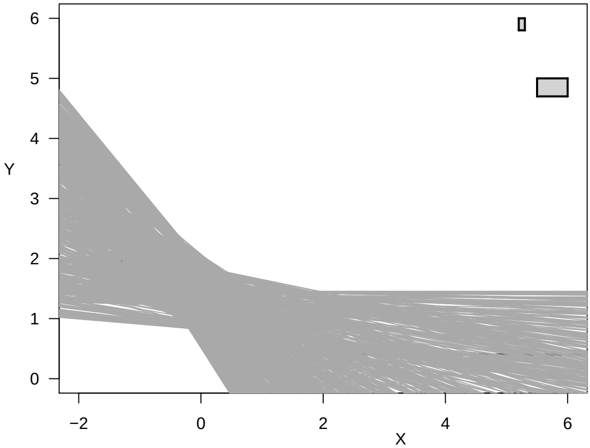

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- further details in: M. Cattaneo, A. Wiencierz (2012). *Likelihood-based Imprecise Regression*. Int. J. Approx. Reasoning 53. 1137-1154.

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- aim: determine the set of undominated functions $\mathcal{U} = \{f \in \mathcal{F} : \underline{r}_{f,(\underline{k}+1)} \leq \overline{q}_{LRM}\}$

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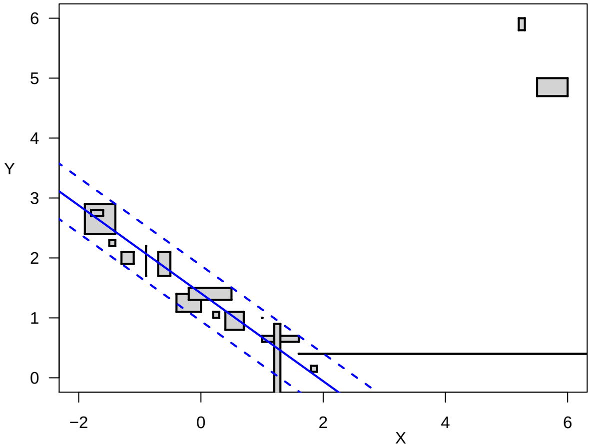
- 1st step: find $\overline{q}_{LRM}$

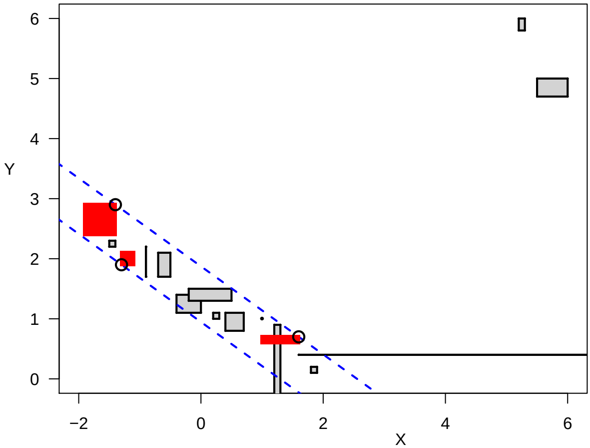
Implementation: Exact algorithm for simple linear LIR

- aim: determine the set of undominated functions $\mathcal{U} = \{f \in \mathcal{F} : \underline{r}_{f,(\underline{k}+1)} \leq \overline{q}_{LRM}\}$
- 1st step: find $\overline{q}_{LRM}$
- $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$ (blue dashed lines) is the thinnest band containing at least $\overline{k}$ imprecise data
- here $\beta = 0.8, \underline{p} = 0.6,$ $n = 17$, and $\overline{k} = 12$



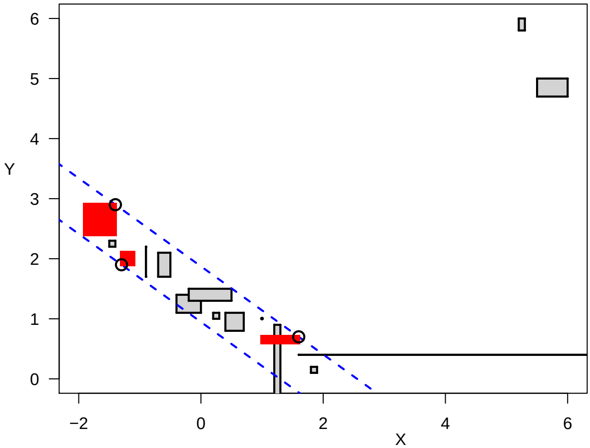
Implementation: Exact algorithm - Part 1

- some of the included $\overline{k}$ imprecise observations touch the borders of $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$ in 3 different points



Implementation: Exact algorithm - Part 1

- some of the included $\overline{k}$ imprecise observations touch the borders of $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$ in 3 different points
- b_{LRM} can be any slope determined by the corresponding corner points of 2 imprecise data or 0



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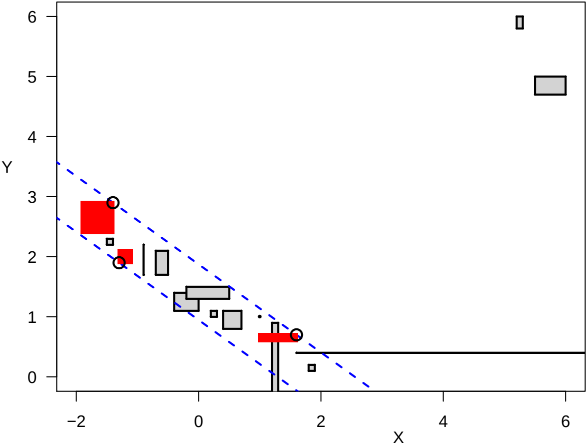

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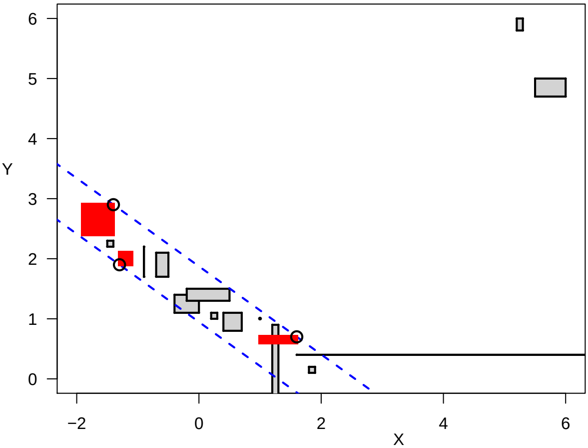

Implementation: Exact algorithm - Part 1

for each $b \in \mathcal{B}$

- consider transformed data $z_i^* = [\underline{z}_i, \overline{z}_i]$ with

$$\underline{z}_i = \begin{cases} \underline{y}_i - b\,\overline{x}_i\,, & b > 0 \\ \underline{y}_i - b\,\underline{x}_i\,, & b \leq 0 \end{cases} \quad \text{and} \quad \overline{z}_i = \begin{cases} \overline{y}_i - b\,\underline{x}_i\,, & b > 0 \\ \overline{y}_i - b\,\overline{x}_i\,, & b \leq 0 \end{cases}$$

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$$\Rightarrow \quad \overline{q}_{LRM} = \frac{1}{2} \min_{(b,j) \in \mathcal{B} \times \{1,\ldots,n-\overline{k}+1\}} \left(\overline{z}_{b,[j]} - \underline{z}_{b,(j)}\right)$$

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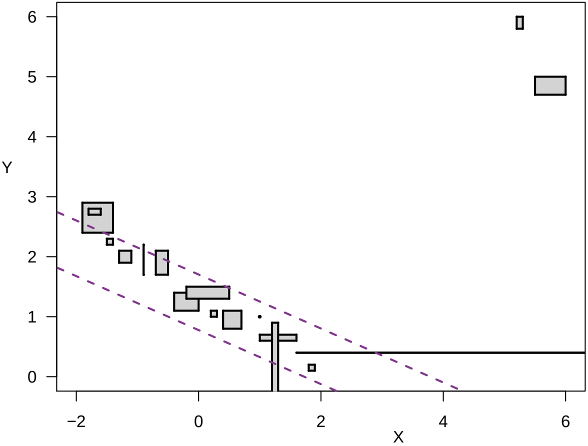

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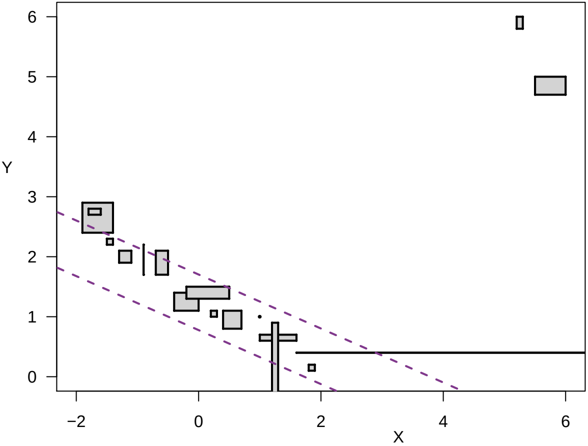

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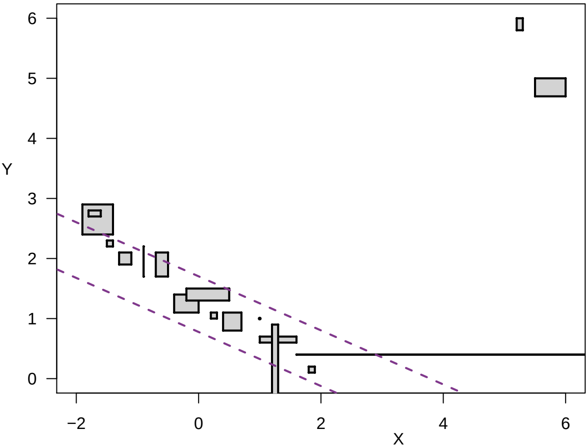

Implementation: Exact algorithm - Result

$$\Rightarrow \quad \mathcal{U} = \left\{ f_{a,b} : b \in \mathbb{R} \text{ and } a \in \bigcup_{j=1}^{n-\underline{k}} [\underline{z}_{b,(\underline{k}+j)} - \overline{q}_{LRM}, \overline{z}_{b,(j)} + \overline{q}_{LRM}] \right\}$$

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- further details in: M. Cattaneo, A. Wiencierz (2012). *On the implementation of LIR: the case of simple linear regression with interval data*. Technical Report 127. Department of Statistics. LMU Munich.

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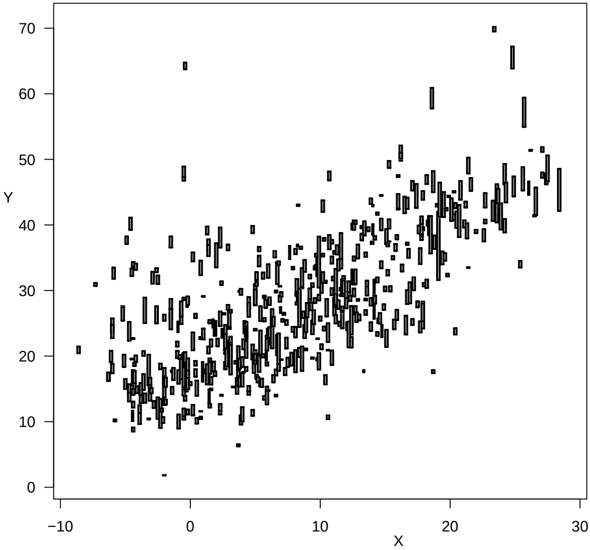
- further tools to summarize and visualize results

Example

- 2-dimensional interval data set of $n = 514$ observations



Example

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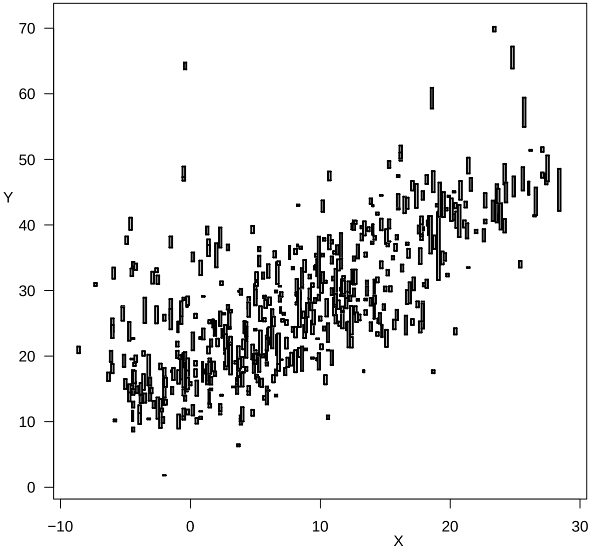

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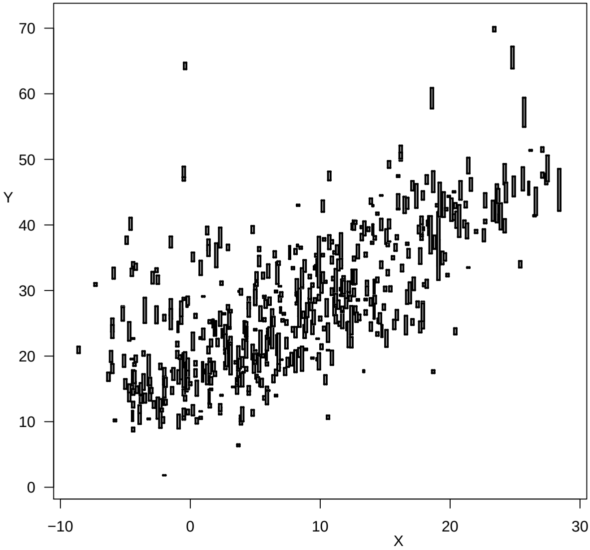

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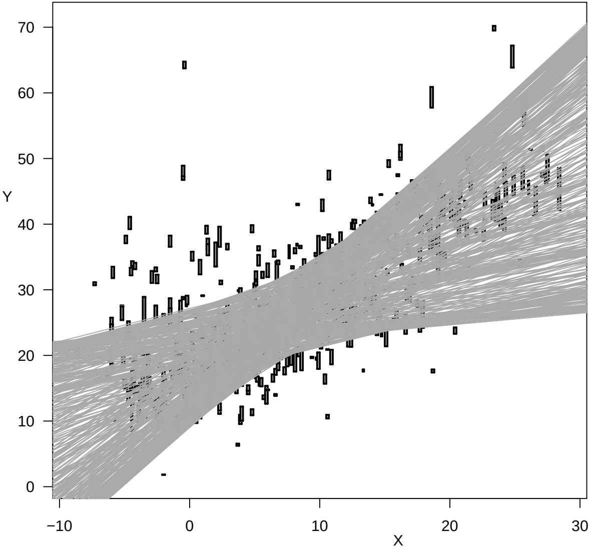

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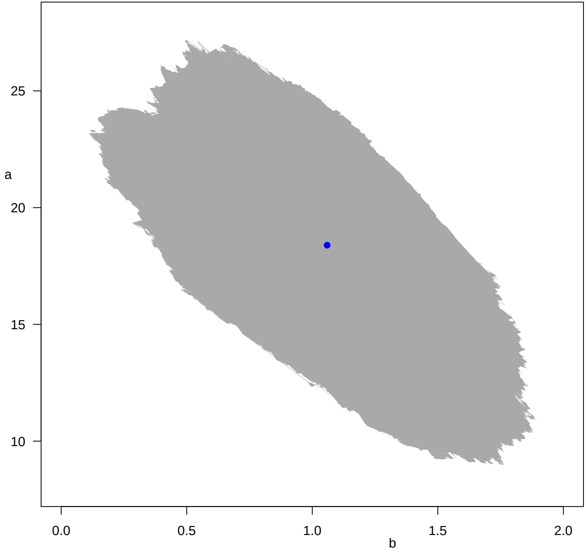

Future work

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