An exact algorithm for Likelihood-based Imprecise Regression in the case of simple linear regression with interval data

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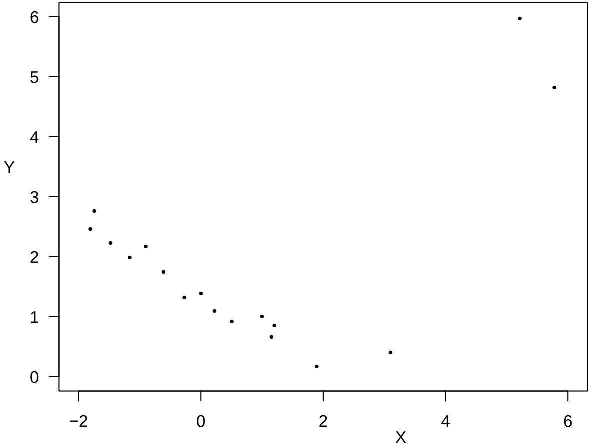

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- simple linear regression:

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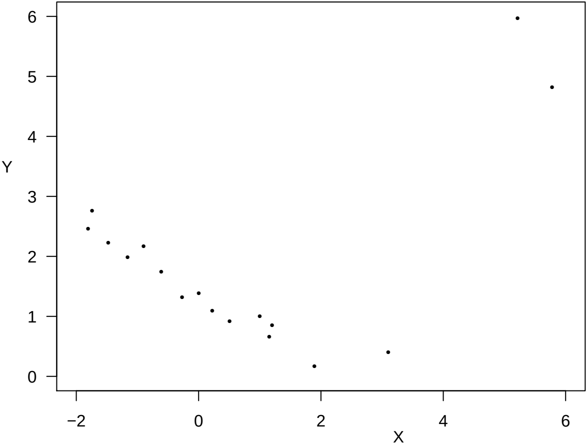

(Simple) linear LIR with interval data

- $(X_1^*, Y_1^*), \ldots, (X_n^*, Y_n^*)$
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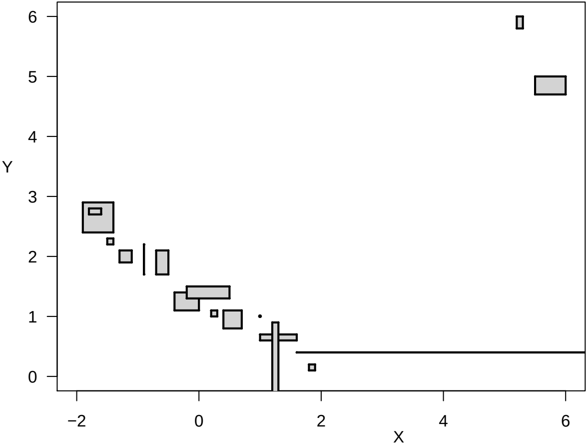

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- with $V_i^* = X_i^* \times Y_i^*$
  $((X_i, Y_i), V_i^*) \overset{\text{i.i.d.}}{\sim} P$
  such that for $\varepsilon \in [0, 1]$
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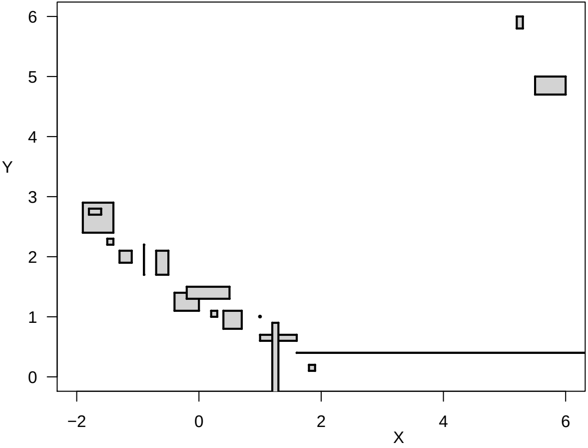

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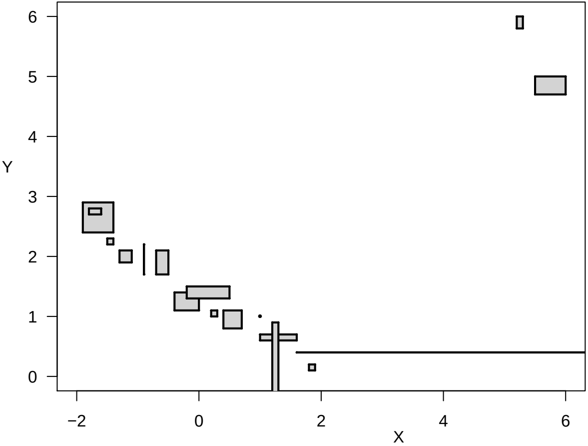

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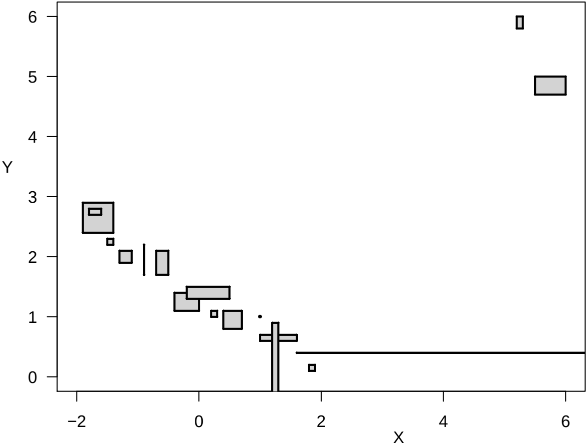
- simple linear regression: $Y = f(X) = a + b\,X$
- p-quantile $Q_{R_f,p}$, with $p \in (0, 1)$, of the distribution of the residuals $R_{f,i} = |Y_i - f(X_i)|$



(Simple) linear LIR with interval data

- imprecise residuals:

$$\underline{r}_{f,i} = \min_{(x,y) \in v_i^*} |y - f(x)|$$

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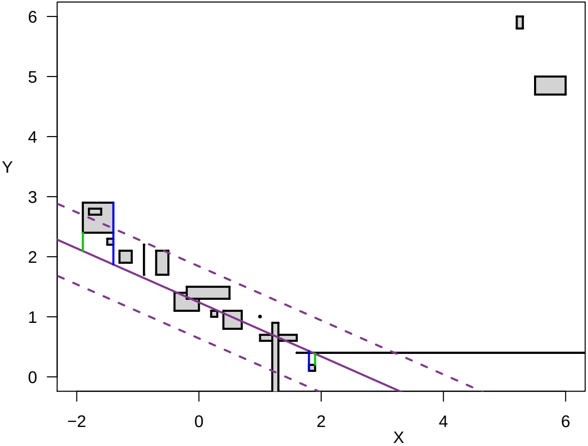

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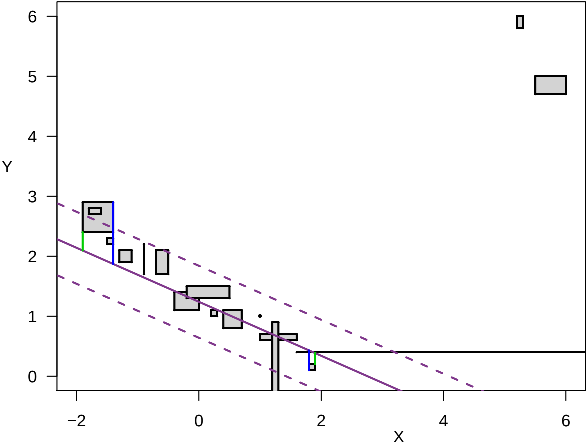

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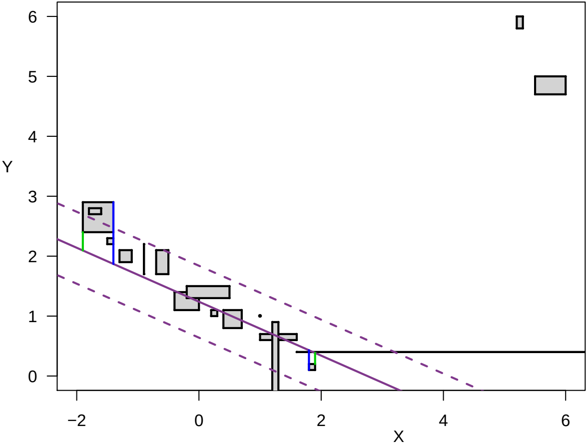
- consider $\mathcal{C}_{f,p,\beta,\varepsilon}$:

  likelihood-based confidence region for $Q_{R_f,p}$ with cutoff point $\beta \in (0,1)$



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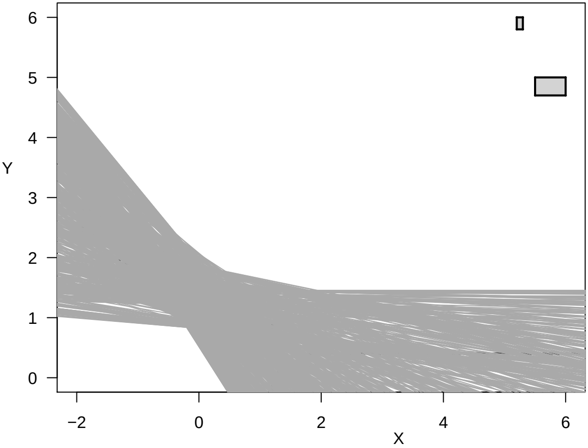
- result $\mathcal{U}$: set of all plausible functions



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- LIR result $\mathcal{U} = \{f \in \mathcal{F} : \underline{r}_{f,(\underline{k}+1)} \leq \overline{q}_{LRM}\}\,,$ where $\overline{q}_{LRM} = \inf_{f \in \mathcal{F}} \overline{r}_{f,(\overline{k})}$

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- further details in: M. Cattaneo, A. Wiencierz (2012). *Likelihood-based Imprecise Regression*. Int. J. Approx. Reasoning 53. 1137-1154.

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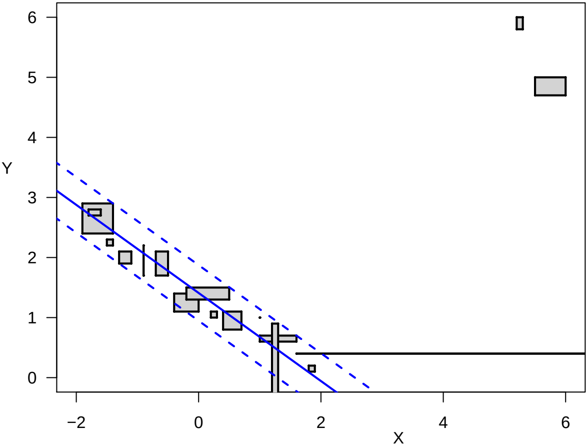
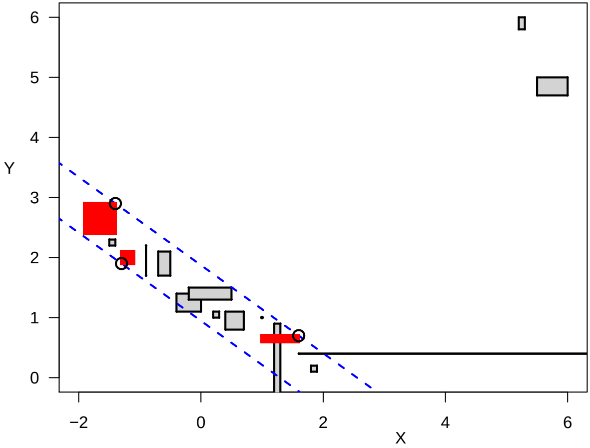
- 1st step: find $\overline{q}_{LRM}$
- $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$ (blue dashed lines) is the thinnest band containing at least $\overline{k}$ imprecise data
- here $\beta = 0.8, p = 0.6,$ $n = 17\,,$ and $\overline{k} = 12$



Implementation: Exact algorithm - Part 1

- some of the included $\overline{k}$ imprecise data touch the border of $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$ in 3 different points



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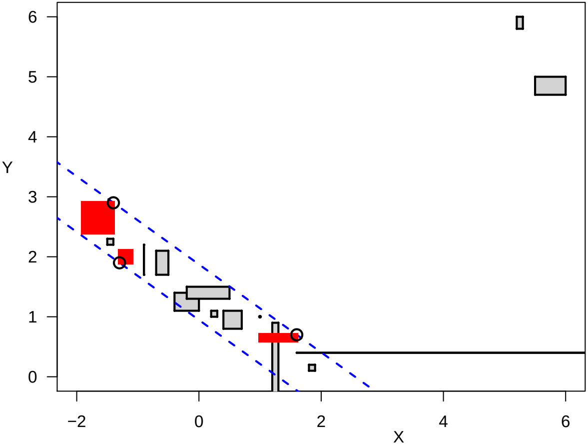

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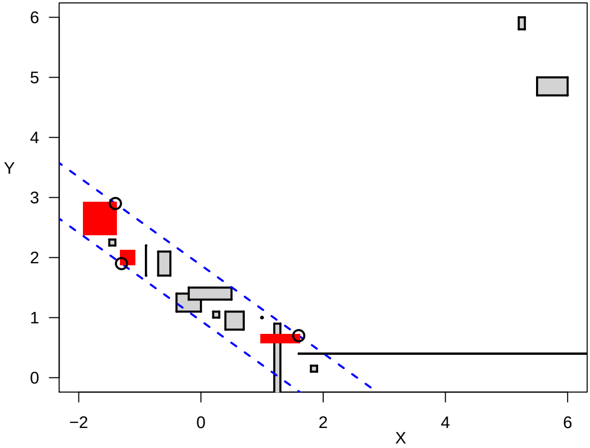

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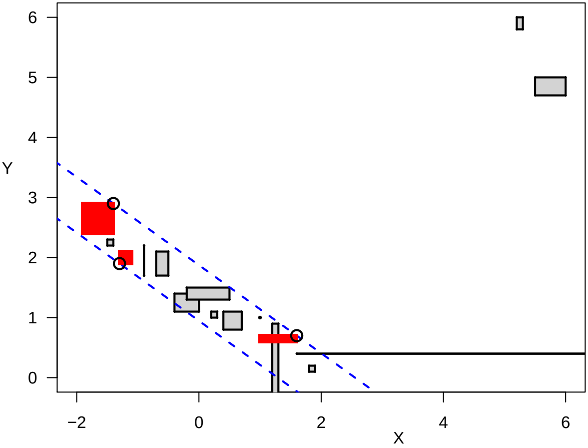
- $\mathcal{B}$: set of all $4\binom{n}{2}+1$ possible values for b_{LRM}
- for each $b \in \mathcal{B}$ find $a_b \in \mathbb{R}$ for which $\overline{r}_{f_{a_b,b},(\overline{k})}$ is minimal



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for each $b \in \mathcal{B}$

- consider transformed data $z^*_{b,i} = [\underline{z}_{b,i}, \overline{z}_{b,i}]$ with

$$\underline{z}_{b,i} = \begin{cases} \underline{y}_i - b\,\overline{x}_i\,, & b > 0 \\ \underline{y}_i - b\,\underline{x}_i\,, & b \leq 0 \end{cases} \quad \text{and} \quad \overline{z}_{b,i} = \begin{cases} \overline{y}_i - b\,\underline{x}_i\,, & b > 0 \\ \overline{y}_i - b\,\overline{x}_i\,, & b \leq 0 \end{cases}$$

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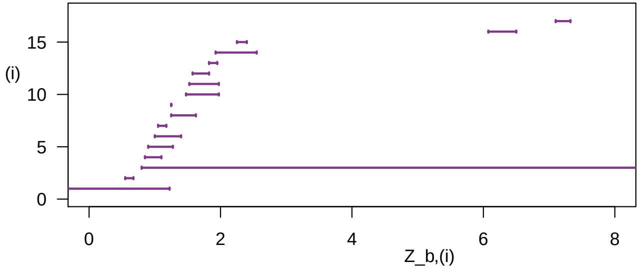
- for example, $z^*_{b,i}$ for $b = -0.25$, ordered by lower endpoint



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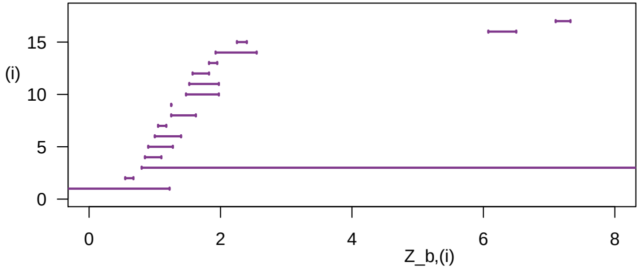

- find the shortest interval $\mathcal{I}_b$ containing at least $\overline{k}$ of the transformed data $z^*_{b,i}$, i.e., determine the shortest of the $n - \overline{k} + 1$ intervals of the form $[\underline{z}_{b,(j)}, \overline{z}_{b,[j]}]$, where $\overline{z}_{b,[j]}$ is the $\overline{k}$th smallest value among the $\overline{z}_{b,i}$ such that $\underline{z}_{b,i} \geq \underline{z}_{b,(j)}$

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for each $b \in \mathcal{B}$

- the length of $\mathcal{I}_b$ corresponds to the width of the closed band around the function $f_{a_b,b}$ containing at least $\overline{k}$ imprecise data

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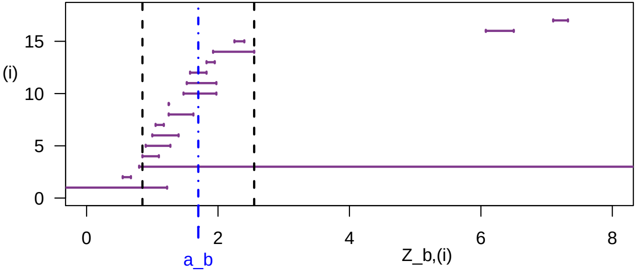

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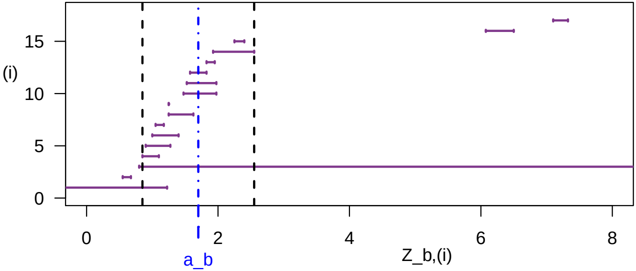
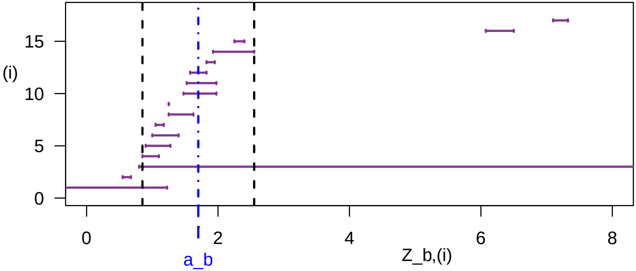


- the associated $\overline{r}_{f_{a_b,b},(\overline{k})}$ corresponds to half of the length of $\mathcal{I}_b$

Implementation: Exact algorithm - Part 1

for each $b \in \mathcal{B}$

- the length of $\mathcal{I}_b$ corresponds to the width of the closed band around the function $f_{a_b,b}$ containing at least $\overline{k}$ imprecise data
- the corresponding intercept a_b is given by the midpoint of the interval $\mathcal{I}_b$
- for example, $b = -0.25$



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then, we obtain $\overline{q}_{LRM}$ by $\quad \overline{q}_{LRM} = \dfrac{1}{2} \min_{(b,j)\in\mathcal{B}\times\{1,\ldots,n-\overline{k}+1\}} \left(\overline{z}_{b,[j]} - \underline{z}_{b,(j)}\right)$

Implementation: Exact algorithm - Part 2

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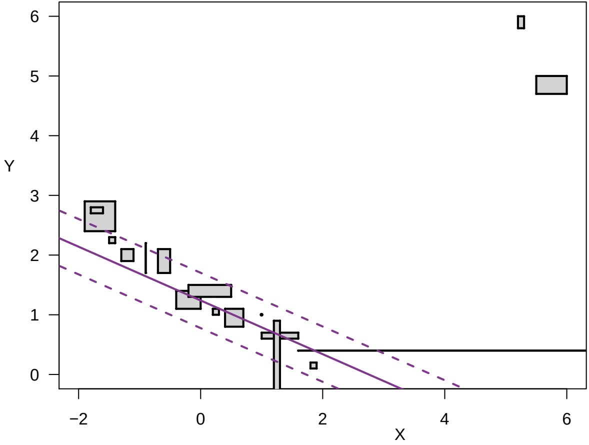

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- for $f \in \mathcal{U}$, the band $\overline{B}_{f,\overline{q}_{LRM}}$ intersects at least $\underline{k} + 1$ data
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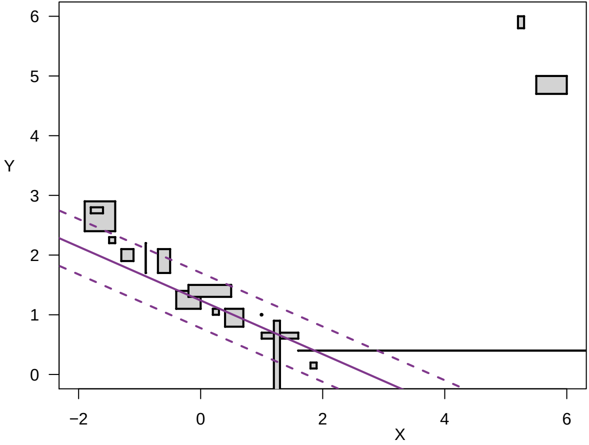
- for $b \in \mathbb{R}$ find all intercept values $a \in \mathbb{R}$, for which $\underline{r}_{f_{a,b},(\underline{k}+1)} \leq \overline{q}_{LRM}$



Implementation: Exact algorithm - Part 2

for a given $b \in \mathbb{R}$

- consider again the transformed data $z^*_{b,i}$

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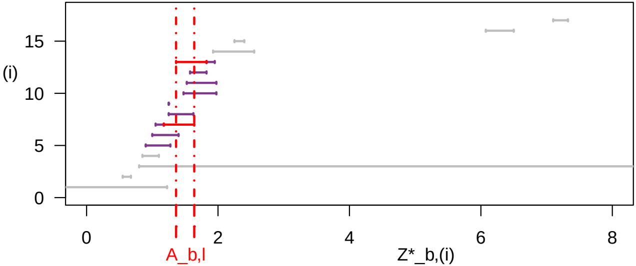

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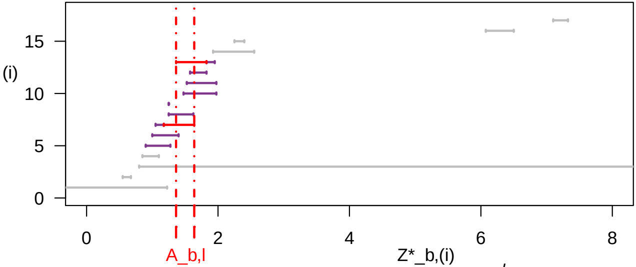

- the union of all $A_{b,l}$ is equivalent to $\mathcal{A}_b = \bigcup_{j=1}^{n-\underline{k}} [\underline{z}_{b,(\underline{k}+j)} - \overline{q}_{LRM}, \overline{z}_{b,(j)} + \overline{q}_{LRM}]$

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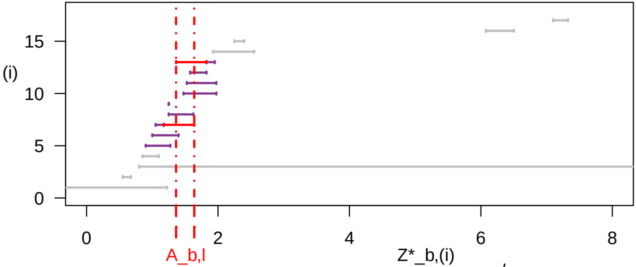

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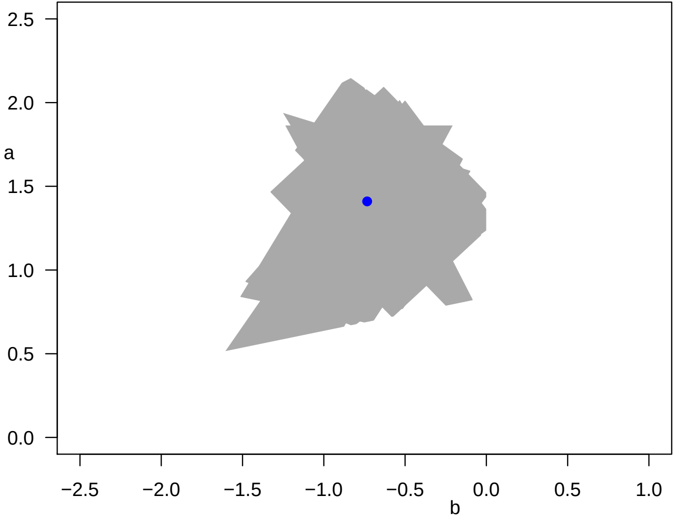
finally, we obtain $\mathcal{U}$ as the set $\{f_{a,b} : b \in \mathbb{R} \text{ and } a \in \mathcal{A}_b\}$

Resulting set of undominated parameters



Implementation of the algorithm in R

Recapitulation: Exact algorithm for simple linear LIR with interval data

- aim: determine the LIR result $\mathcal{U}$, i.e., the set of all functions that are plausible relations of X and Y in the light of the imprecise observations

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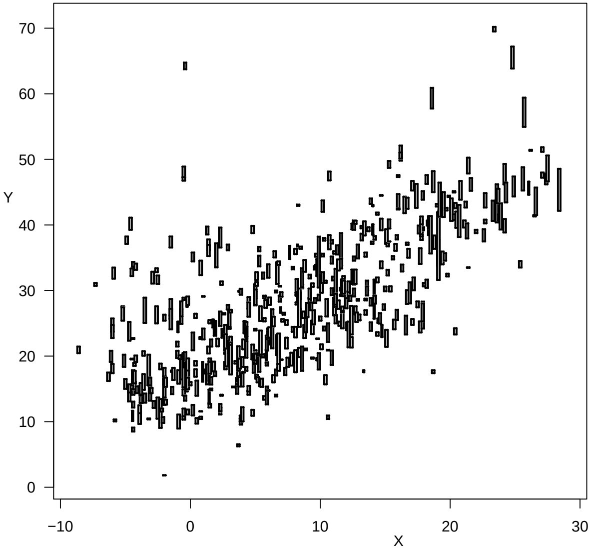
- further tools to summarize and visualize results

Example - Data set

- 2-dimensional interval data set of $n = 514$ observations



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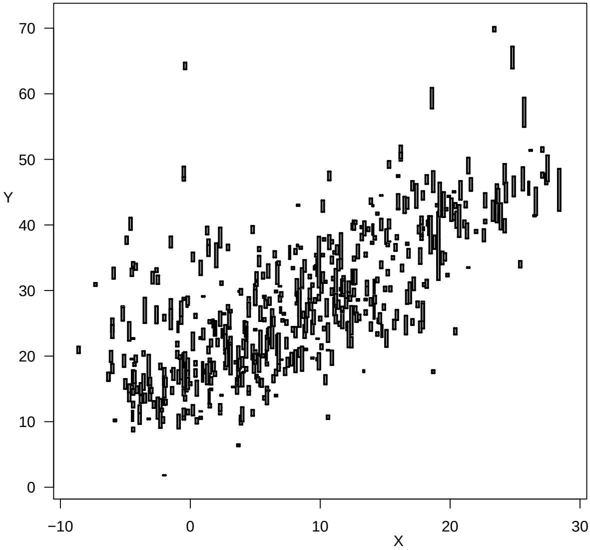

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- 2-dimensional interval data set of $n = 514$ observations
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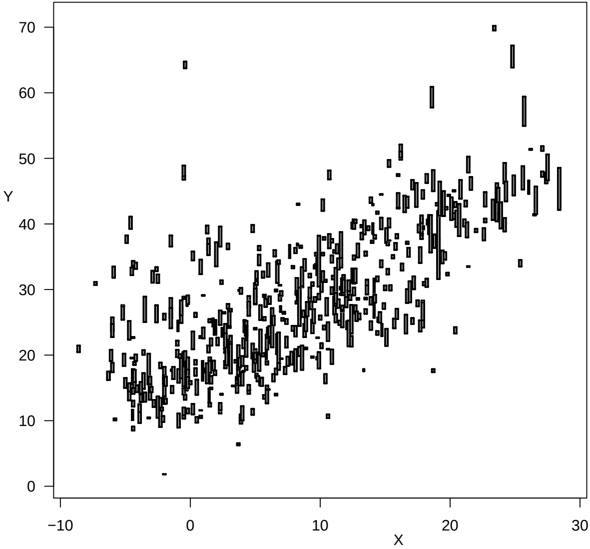

Example - R code

```
library(linLIR)
data(pm10)
pm.idf <- idf.create(pm10, var.labels=c("X","Y"))
pm.lir <- s.linlir(pm.idf, p = 0.5, bet=0.26, epsilon = 0)
summary(pm.lir)
```

```
Ranges of parameter values of the undominated functions:
intercept of f in [8.977766,27.18173]
slope of f in [0.11,1.898]

Bandwidth:  10.79207

Estimated parameters of the function f.lrm:
intercept of f.lrm:  18.39075
slope of f.lrm:  1.059185

Number of observations:  514

LIR settings:
p:  0.5 beta:  0.26 epsilon:  0 k.l:  238 k.u:  276
confidence level of each confidence interval:  90.61 %
```

Example - Undominated regression functions

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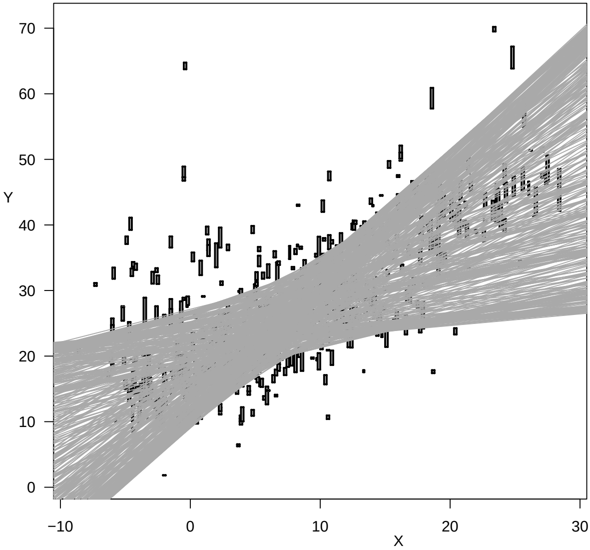
- obtained set of all undominated regression functions



Example - Undominated parameters

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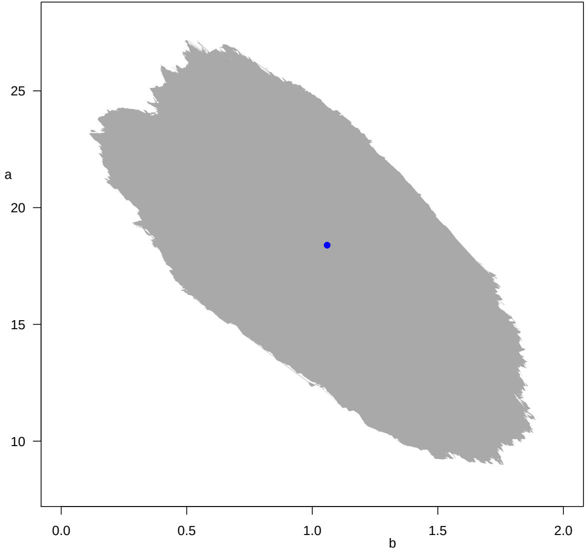
- obtained set of parameters



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- future work: generalize algorithm to multiple linear regression