Likelihood-based inference for probabilistic graphical models: Some preliminary results

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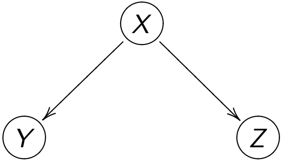
example

$X, Y, Z \in \{0, 1\}$



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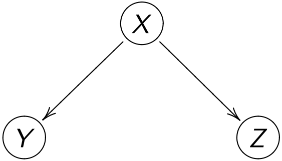

data:

X	Y	Z	#
0	0	0	15
0	0	1	25
0	1	0	7
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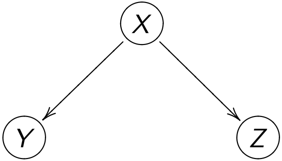

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inference about $P(X = 1 \mid Y = 1,\ Z = 1)$:

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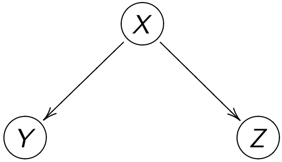

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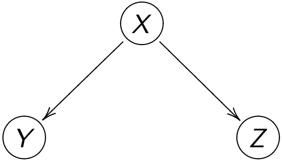

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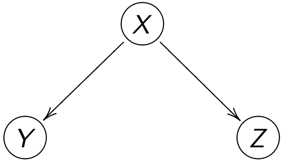

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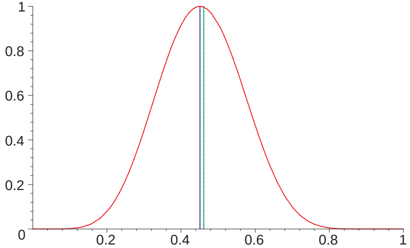

example $\times$ 100

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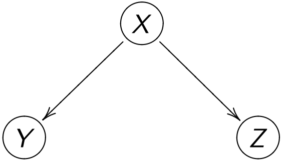

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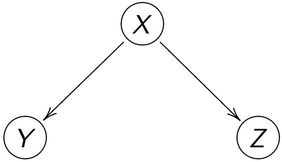

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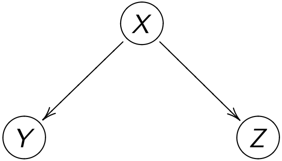

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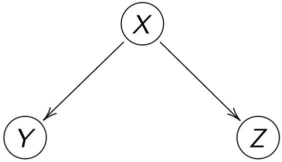

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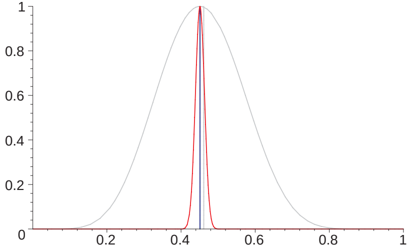

likelihood

- **profile likelihood function for $P(X = 1 \mid Y = 1,\, Z = 1)$:**

$$\boxed{lik(p) \propto \max_{\substack{bn \in \mathcal{BN}\,: \\ P_{bn}(X=1 \mid Y=1,\, Z=1)=p}} P_{bn}(\text{data})} \qquad \text{for all } p \in [0,1],$$

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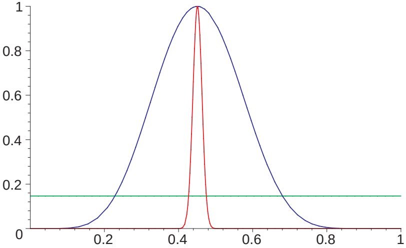

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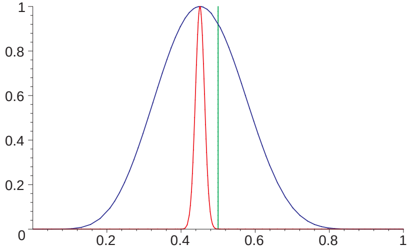

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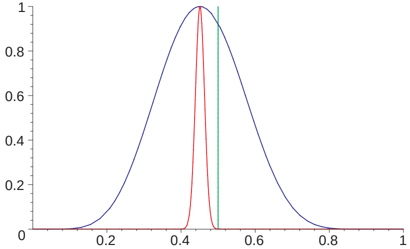

- **future research:** generalization to larger classes of problems and consideration of the uncertainty about the graph