interpretations of probability

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frequentist

subjective

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$[\underline{P}(A), \overline{P}(A)]$ can be used as a description of what we have learned about $P(A)$

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foundations of statistics

frequentist approach	likelihood approach	Bayesian approach
empirical	empirical	personalistic
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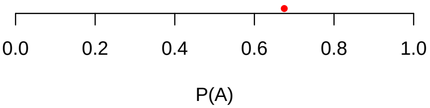
precise probability



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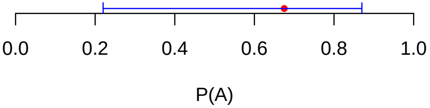

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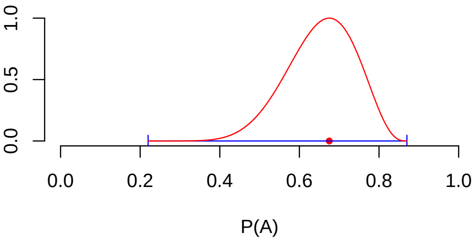

example (Wilson, ISIPTA '01)

Let $P(Y = 0) = P(Y = 1) = 0.5$,
and let $X_1, X_2, \ldots, X_{100} \in \{0, 1\}$ be i.i.d. conditional on Y
with $\quad P(X_i = 1 \mid Y = 0) = 0.5 \quad$ and $\quad 0.1 \leq P(X_i = 1 \mid Y = 1) \leq 0.6$

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After having observed the realizations of $X_1, X_2, \ldots, X_{100}$ with mean 0.2, we would expect the conditional distribution of Y to be concentrated on 1 (since $Y = 1$ is compatible with the observations, while $Y = 0$ is not), but when we update the model by means of "regular extension", we almost obtain complete ignorance about the value of Y (the posterior interval probability of $Y = 0$ is approximately [0.000000004, 0.999999]).

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This interval probability is the support of the density function of the conditional "fuzzy probability" of $Y=0$, which is concentrated toward 0, in agreement with the intuition that the conditional distribution of Y should be concentrated on 1.