Imprecise probability for statistical problems: is it worth the candle?

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introduction

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- theoretical perspective **and** pragmatical perspective (application to the "fundamental problem of practical statistics")

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- **data**: $\sum_{i=1}^{n} X_i = p$
- **quantity of interest**: $P_\theta(\sum_{i=n+1}^{n+m} X_i = r) = \binom{r}{m} \theta^r (1 - \theta)^s$

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classical approach

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- confidence interval for $\binom{r}{m}\,\theta^r\,(1-\theta)^s$ is more **difficult**

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$$\textbf{Bayesian approach} \xrightarrow{\text{calibration, no prior}} \textbf{classical approach}$$

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- (imprecise) expectation **of** $\binom{r}{m}\, \theta^r\, (1 - \theta)^s$ analytically or numerically, but is neither a point estimate nor a confidence/credible interval

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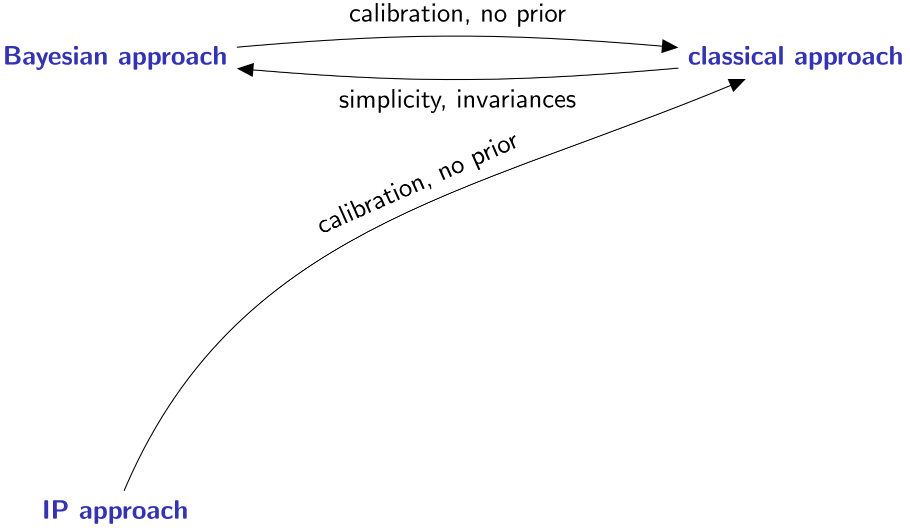
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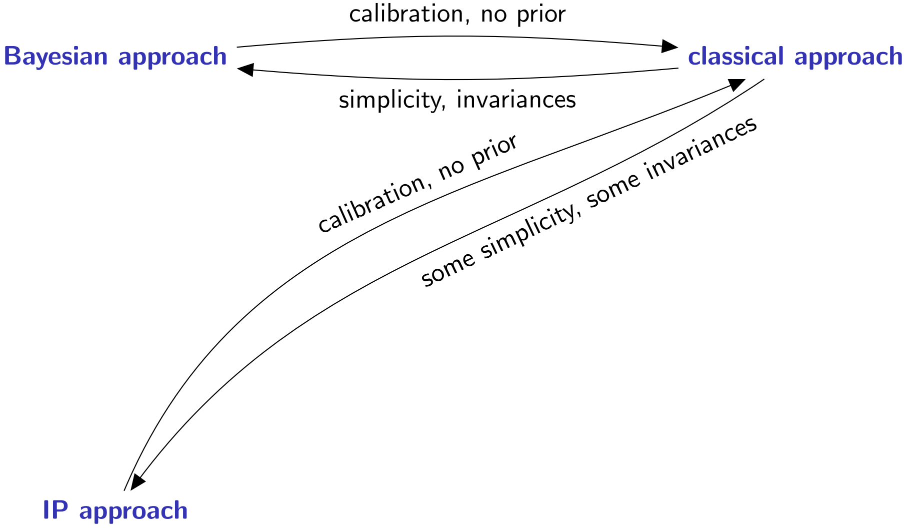
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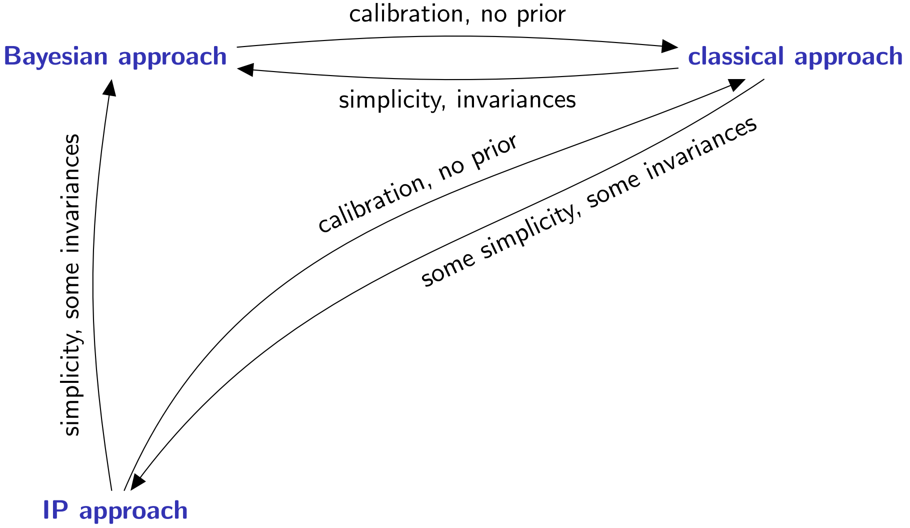
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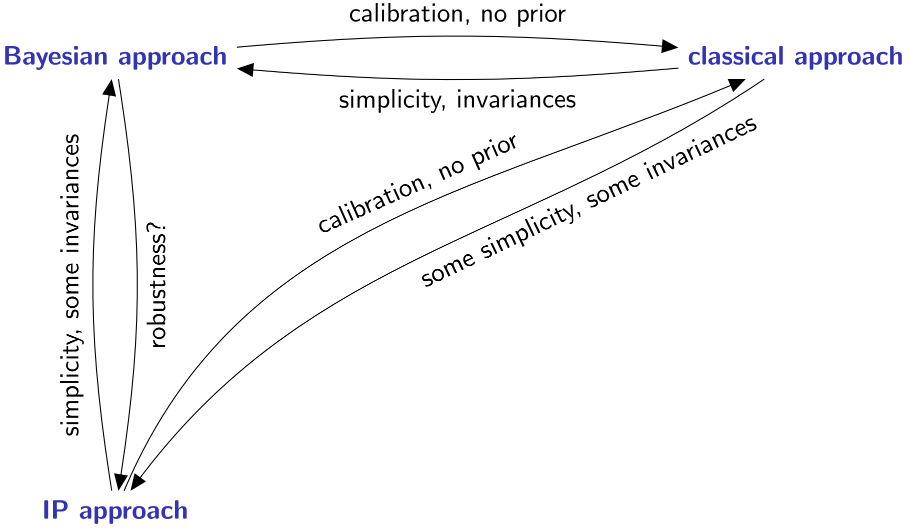
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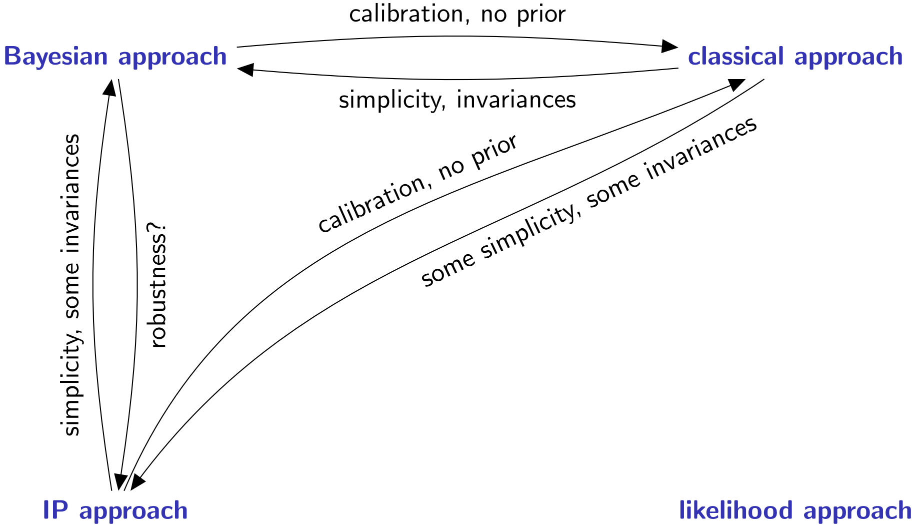
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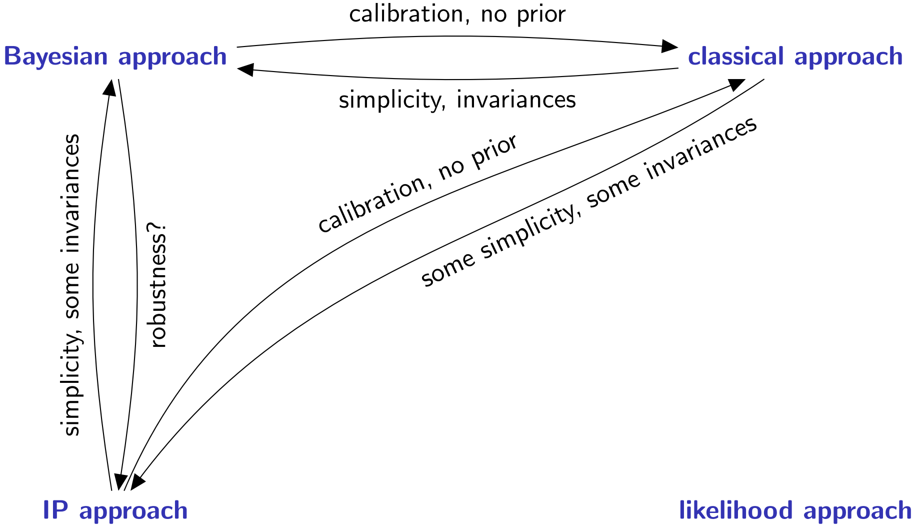
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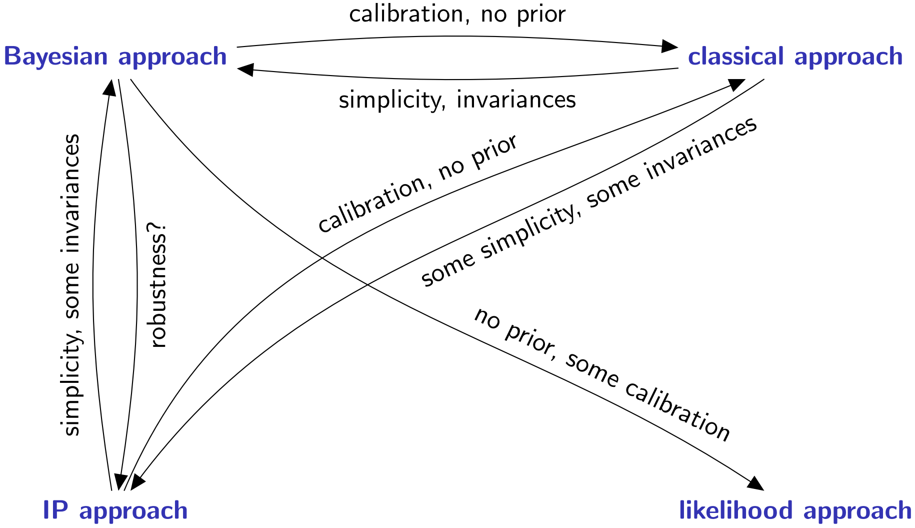
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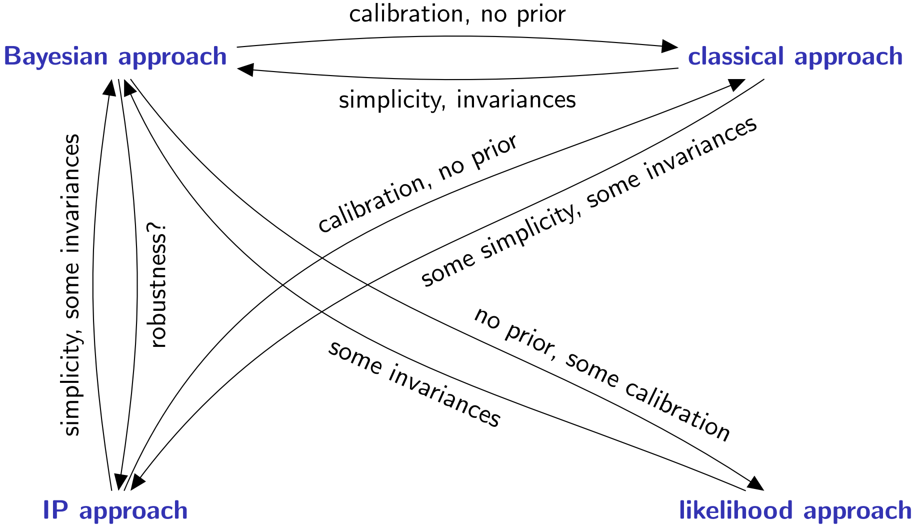
Bayesian approach
classical approach
IP approach
likelihood approach
calibration, no prior
simplicity, invariances
simplicity, some invariances
robustness?
calibration, no prior
some simplicity, some invariances
no prior, some calibration
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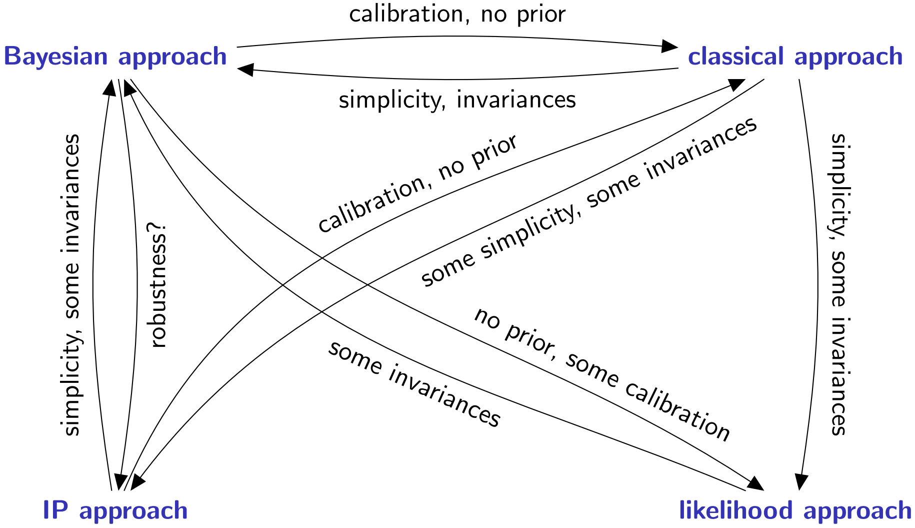
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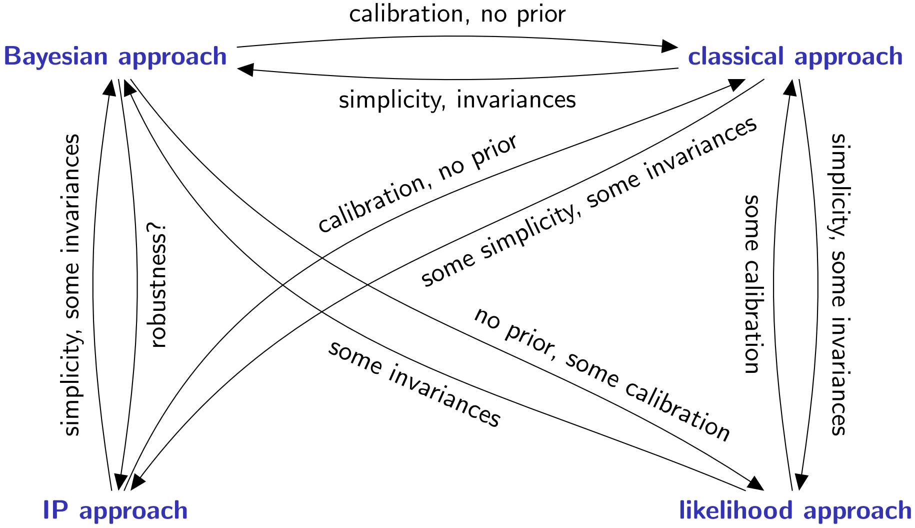
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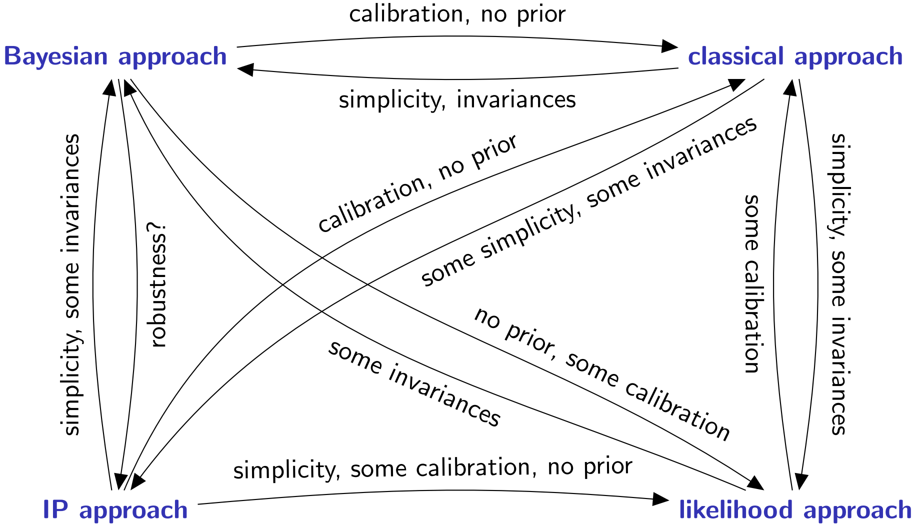
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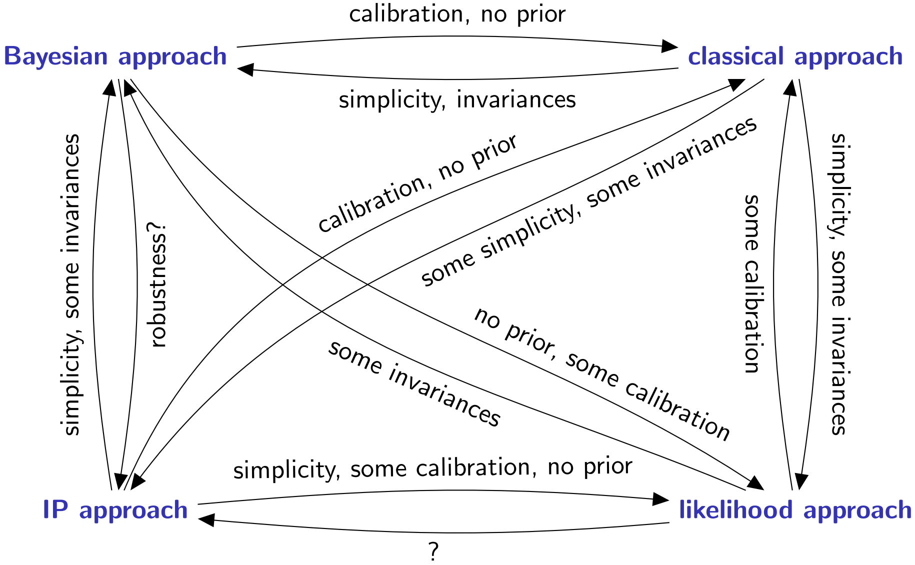
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  - imprecise expectations are often **misinterpreted** as confidence/credible intervals
  - choosing the amount of **imprecision** in prior lower/upper previsions is particularly difficult
- the likelihood approach to statistics seems to be a better **compromise** between the Bayesian and classical ones