

probabilistic graphical models

X, Y, Z categorical random variables

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$$\begin{aligned} P(X = x, Y = y, Z = z) &= \\ &= P(X = x) P(Y = y | X = x) P(Z = z | X = x, Y = y) \end{aligned}$$

probabilistic graphical models

X, Y, Z categorical random variables

Y and Z independent conditional on X

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probabilistic graphical models

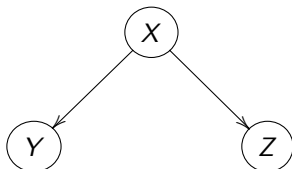
X, Y, Z categorical random variables

Y and Z independent conditional on X

$$P(X = x, Y = y, Z = z) =$$

$$= P(X = x) P(Y = y | X = x) P(Z = z | X = x, Y = y)$$

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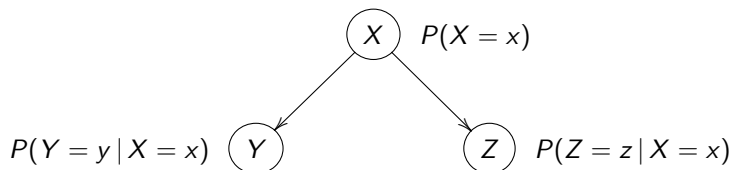


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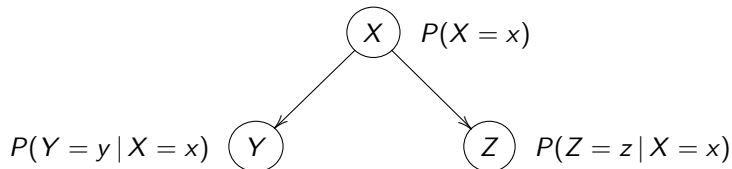


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Y and Z independent conditional on X

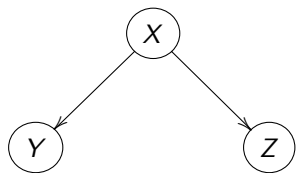
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local Markov condition: each variable is conditionally independent of its non-descendants, given its parents

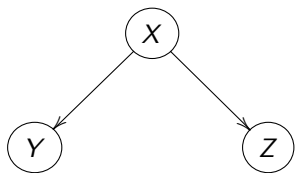
example

$X, Y, Z \in \{0, 1\}$



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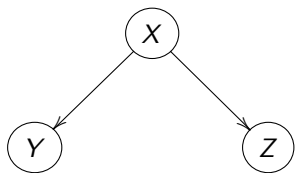


data:

X	Y	Z	#
0	0	0	15
0	0	1	25
0	1	0	7
0	1	1	5
1	0	0	6
1	0	1	35
1	1	0	3
1	1	1	4
			100

example

$X, Y, Z \in \{0, 1\}$



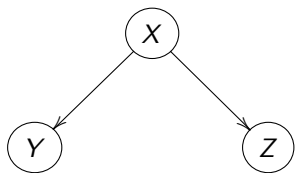
inference about $P(X = 1 | Y = 1, Z = 1)$:

data:

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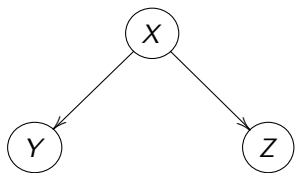
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inference about $P(X = 1 | Y = 1, Z = 1)$:

- ▶ ML estimate: 0.45

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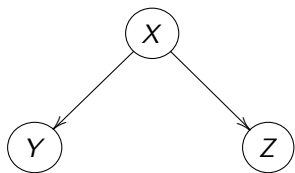
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inference about $P(X = 1 | Y = 1, Z = 1)$:

- ▶ ML estimate: 0.45
- ▶ Bayesian estimate with uniform priors: 0.46

example

$X, Y, Z \in \{0, 1\}$

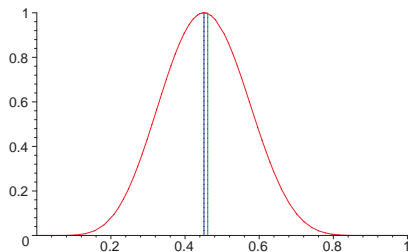


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0	0	1	25
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			100

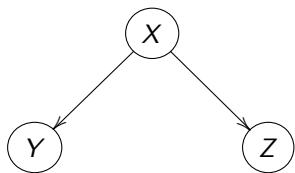
inference about $P(X = 1 | Y = 1, Z = 1)$:

- ▶ ML estimate: 0.45
- ▶ Bayesian estimate with uniform priors: 0.46
- ▶ profile likelihood function:



example $\times 100$

$X, Y, Z \in \{0, 1\}$



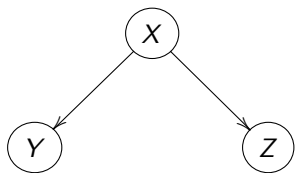
inference about $P(X = 1 | Y = 1, Z = 1)$:

data:

X	Y	Z	#
0	0	0	1500
0	0	1	2500
0	1	0	700
0	1	1	500
1	0	0	600
1	0	1	3500
1	1	0	300
1	1	1	400
			10000

example $\times 100$

$X, Y, Z \in \{0, 1\}$



data:

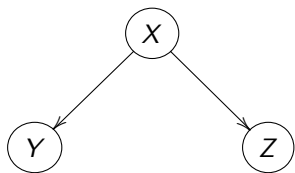
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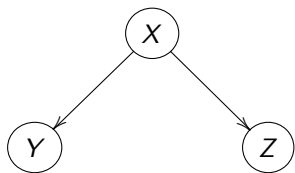
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inference about $P(X = 1 | Y = 1, Z = 1)$:

- ▶ ML estimate: 0.45
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example $\times 100$

$X, Y, Z \in \{0, 1\}$

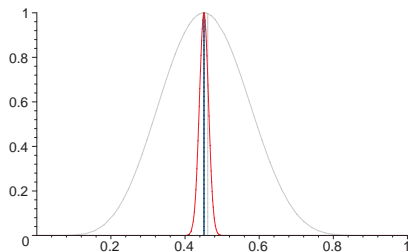


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X	Y	Z	#
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0	1	0	700
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1	0	0	600
1	0	1	3500
1	1	0	300
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foundations of statistics

frequentist approach

empirical
repeated-sampling

likelihood approach

empirical
conditional

Bayesian approach

personalistic
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can be interpreted as an **imprecise probability** approach:

(profile) likelihood function =: membership function of fuzzy probability

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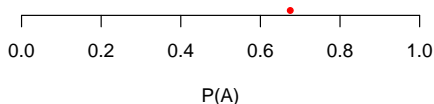


can be interpreted as an **imprecise probability** approach:

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generalizations:

precise probability



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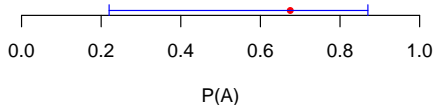
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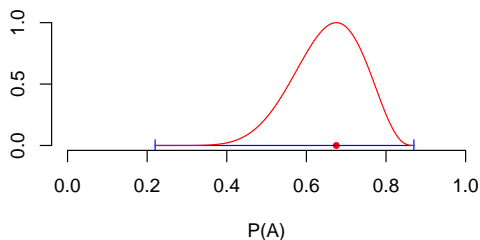
precise probability



interval probability



fuzzy probability



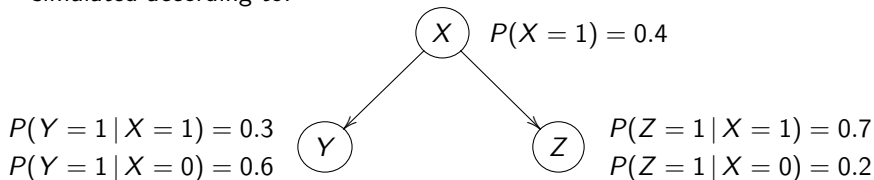
training data

X	Y	Z	#
0	0	0	21
0	0	1	6
0	1	0	30
0	1	1	7
1	0	0	9
1	0	1	15
1	1	0	5
1	1	1	7
			100

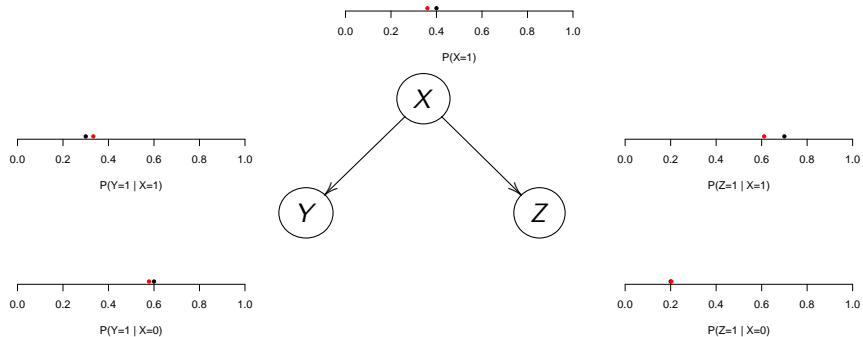
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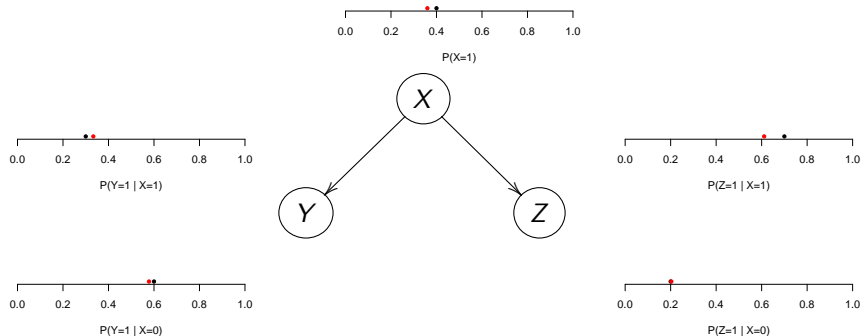
simulated according to:



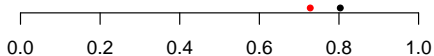
Bayesian network via MLE



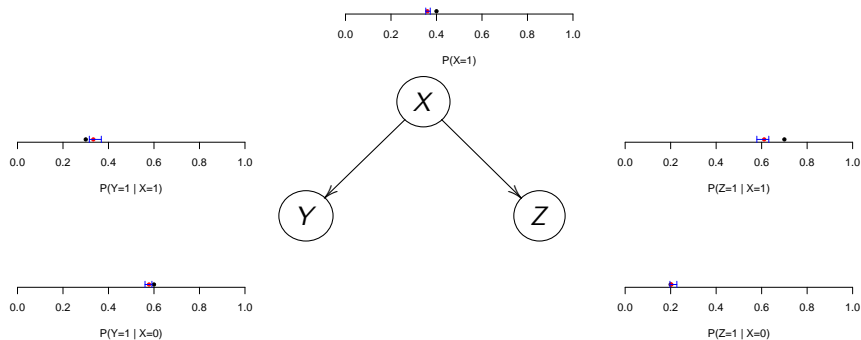
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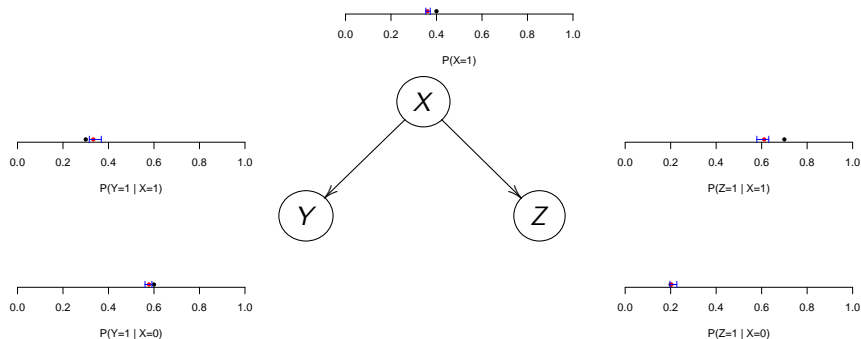
$\Rightarrow P(X = 1 | Y = 0, Z = 1) :$



credal network via IDM (with $s = 2$)



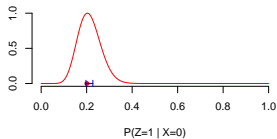
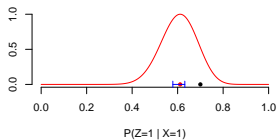
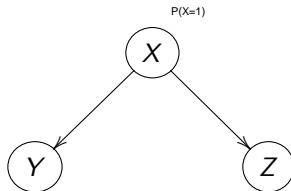
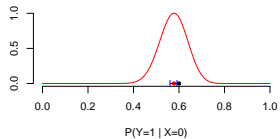
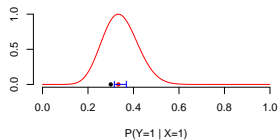
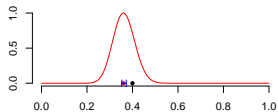
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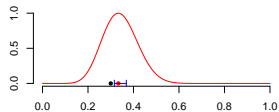
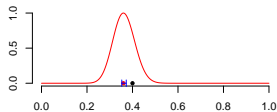
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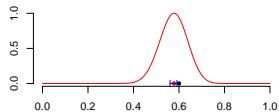
hierarchical network



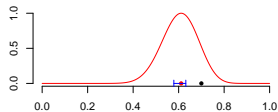
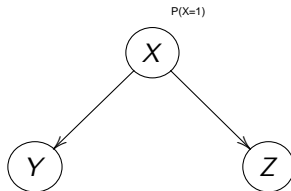
hierarchical network



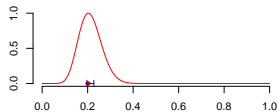
$P(Y=1 | X=1)$



$P(Y=1 | X=0)$



$P(Z=1 | X=1)$



$P(Z=1 | X=0)$

$\Rightarrow P(X=1 | Y=0, Z=1) :$

