Robust regression with imprecise data

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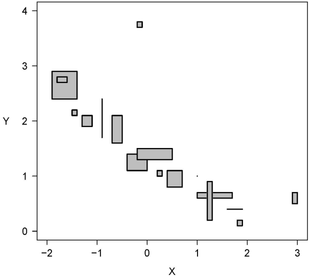
17 November 2011

imprecise data

example data set:



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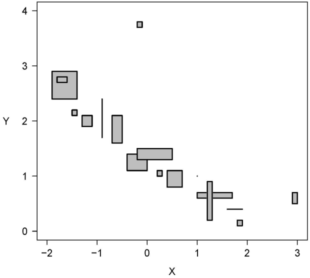

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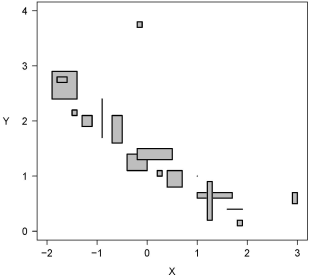

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  - midpoint regression: represent the observed imprecise data by *few precise values* (e.g., intervals by midpoint and length), and apply standard regression methods to those values (see, e.g., Domingues et al., 2010)

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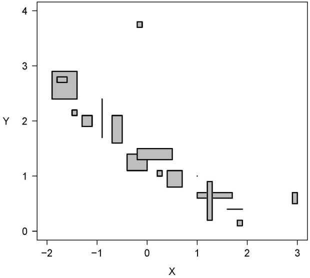

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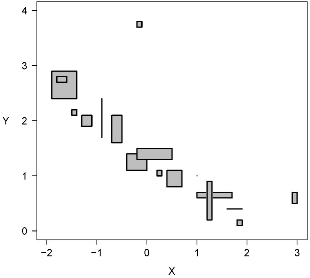

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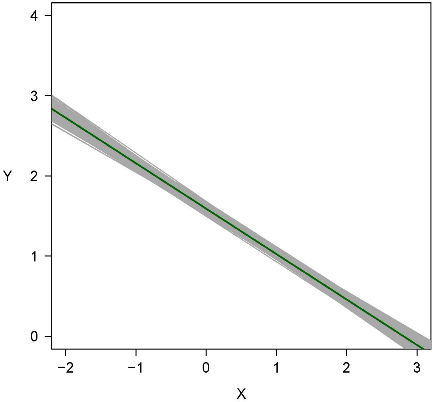
- LIR (Likelihood-based Imprecise Regression): new regression method directly applicable to imprecise data (Cattaneo and Wiencierz, 2011*a*,*b*)

linear regression for the example data set

midpoint regression (green and violet lines) and set of precise regressions (set of gray lines):



least squares

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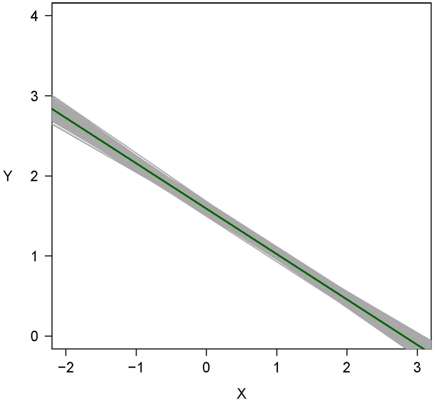

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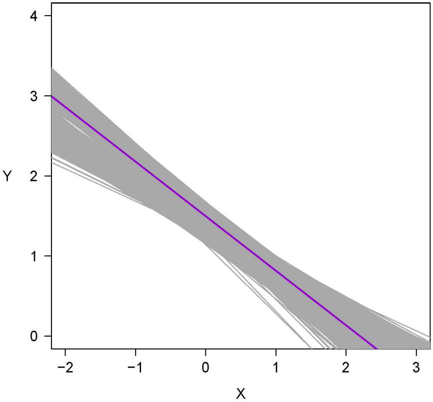

least median of squares

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- the observed (imprecise) data $V_1^* = A_1, \ldots, V_n^* = A_n$ induce the (normalized) likelihood function $lik : \mathcal{P} \to [0, 1]$ with

$$lik(P) = \frac{P(V_1^* = A_1, \ldots, V_n^* = A_n)}{\max_{P' \in \mathcal{P}} P'(V_1^* = A_1, \ldots, V_n^* = A_n)}$$

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- for each function $f \in \mathcal{F}$, the quantiles of the distribution of the absolute residuals $R_{f,i}$ can be estimated even under the nonparametric model $\mathcal{P}$
- the regression problem can be interpreted as the *minimization of the p-quantile* of the distribution of the absolute residuals $R_{f,i}$ (where $p \in (0, 1)$ is fixed)

generalized LQS regression

- likelihood-based confidence interval for the p-quantile of the distribution of the absolute residuals $R_{f,i}$ (where $Q_f(P)$ is the interval of all p-quantiles of $R_{f,i}$ under P, and $\beta \in (0,1)$ is fixed):

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- when the observed data are in fact precise, f_{LRM} corresponds to the LQS (Least Quantile of Squares) estimate with quantile $\frac{\overline{k}}{n}$
- f_{LRM} can be computed by generalizing the algorithm of Rousseeuw and Leroy (1987)

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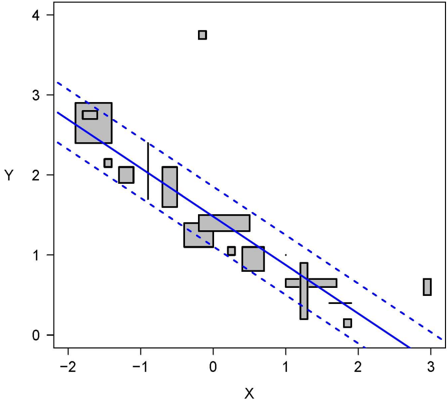
- *complex uncertainty*, consisting of two kinds of uncertainty:
  - statistical uncertainty: decreases when n increases (reflected by the spread between $\frac{\underline{k}+1}{n}$ and $\frac{\overline{k}}{n}$)
  - indetermination: unavoidable under such weak assumptions (reflected by the difference between *containing* and *intersecting* imprecise data)

LIR analysis of the example data set

$$n = 17, \quad \varepsilon = 0, \quad p = 0.5, \quad \beta = 0.5 \quad \Rightarrow \quad \overline{k} = 11,$$



f_{LRM} and $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$

LIR analysis of the example data set

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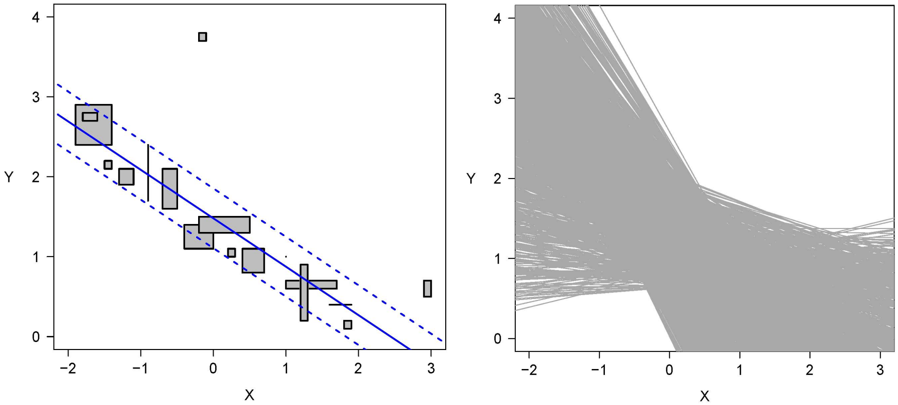

f_{LRM} and $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$ undominated lines ($\overline{k} = 11$, $\underline{k} = 6$)

other values for β and ϵ

$$n = 17, \quad \varepsilon = 0, \quad p = 0.5, \quad \beta = 0.8 \quad \Rightarrow \quad \overline{k} = 10, \quad \underline{k} = 7$$
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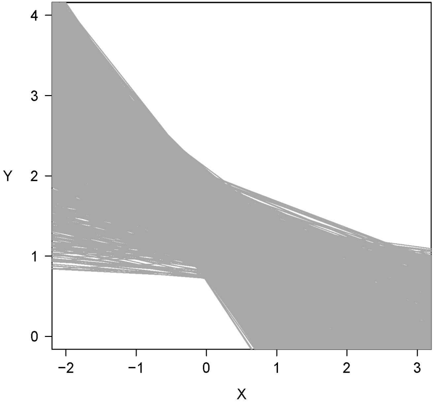

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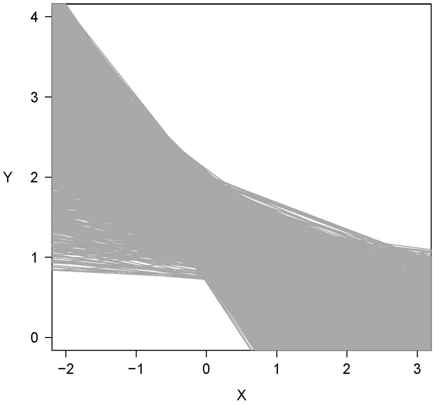

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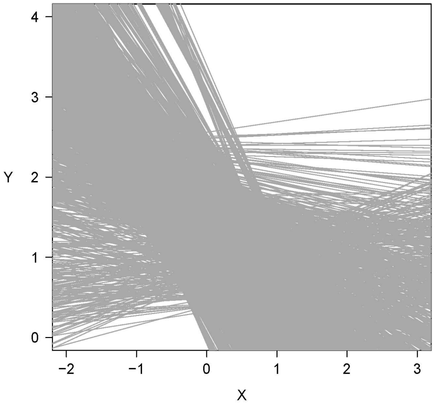

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precise data (midpoints of the example data set)

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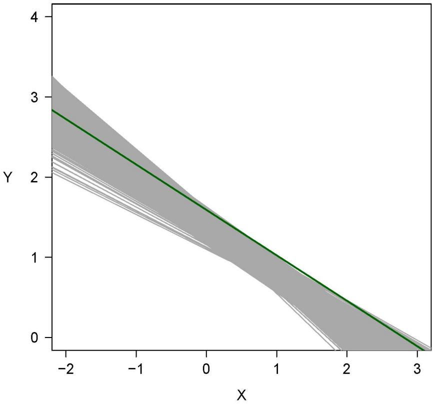

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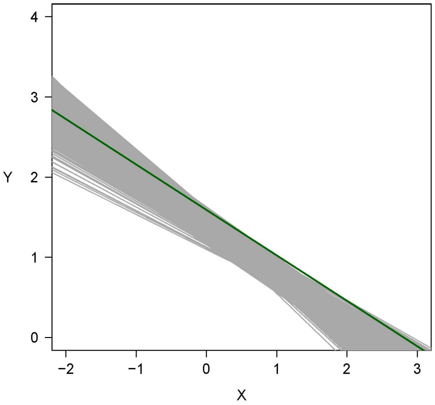

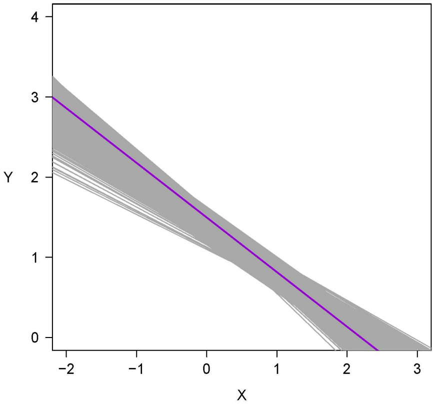

example with social survey data

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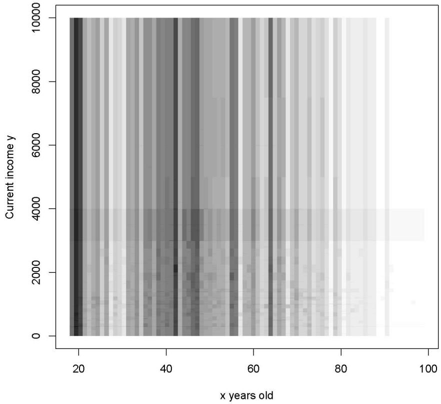
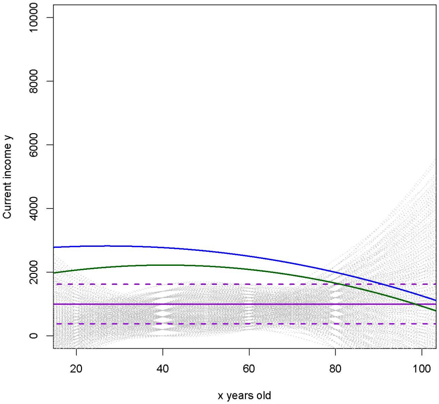
- in 4 different data situations, f_{LRM} (violet solid line, with $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$ represented by the violet dashed lines) and the undominated functions (gray dotted curves) are compared with the results of the least squares midpoint regressions with upper income limit 15000 (blue curve) or 10000 (green curve)

original data

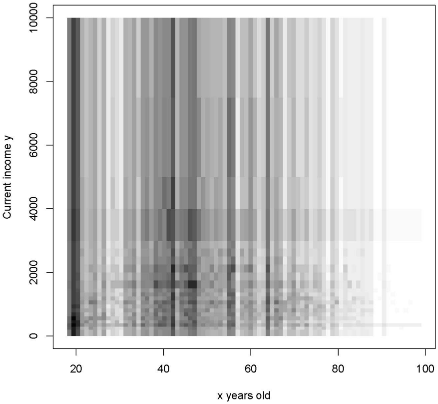
- age data: 3457 “precise” (in years: 83 classes), 12 missing
- income data: 2406 precise, 381 categorized (22 classes), 682 missing

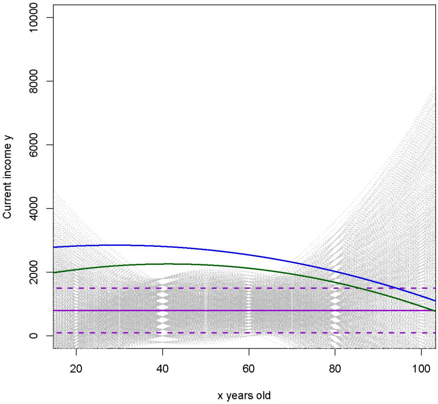




categorized income data

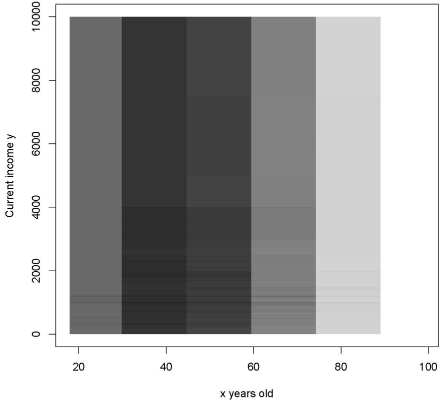
- age data: 3457 “precise” (in years: 83 classes), 12 missing
- income data: 2787 categorized (22 classes), 682 missing

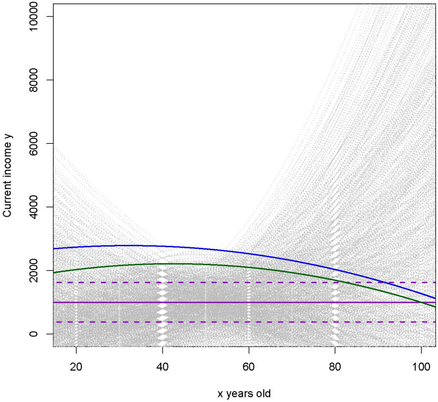




categorized age data

- age data: 3457 categorized (6 classes), 12 missing
- income data: 2406 precise, 381 categorized (22 classes), 682 missing





categorized age and income data

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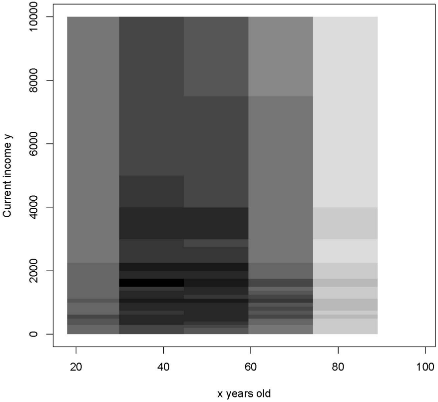
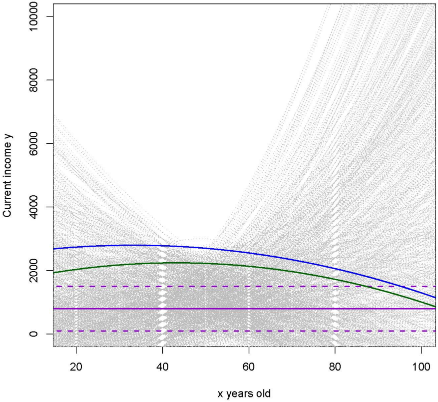
- income data: 2787 categorized (22 classes), 682 missing





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  - consider the minimization of other properties of the distribution of the absolute residuals (besides the quantiles), in order to increase the *efficiency* of the method (e.g., generalized LTS regression)

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