On the Robustness of Imprecise Probability Methods

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- the robustness of the conventional methods refers to the arbitrariness in the choice of $P(A)$, while the robustness of the IP methods refers to the arbitrariness in the choices of $\underline{P}(A)$ and $\overline{P}(A)$

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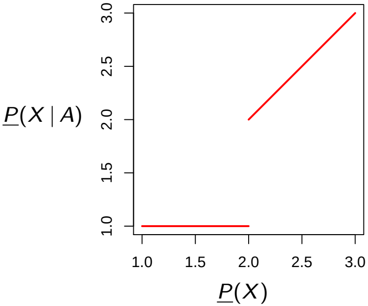

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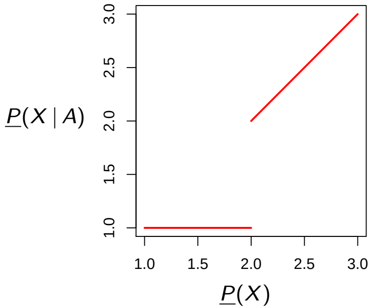

- by contrast, updating of precise probabilities is continuous

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method:	precise	$\checkmark$	$\checkmark$
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method:	precise	✓	✓
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- the gain in robustness is obtained by allowing the methods to be imprecise, and not necessarily by basing them on IP models

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- this is particularly important in statistics, where the (higher) robustness of the IP approach could have been one of the few general advantages over the Bayesian approach
- nevertheless, IP models (with the sensitivity analysis interpretation) can be useful as a tool for studying the robustness of Bayesian methods