

A Generalization of Credal Networks

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foundations of statistics

frequentist approach

empirical
repeated-sampling

likelihood approach

empirical
conditional

Bayesian approach

personalistic
conditional

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can be interpreted as an **imprecise probability** approach:

(profile) likelihood function =: membership function of fuzzy probability

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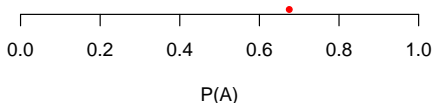


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generalizations:

precise probability



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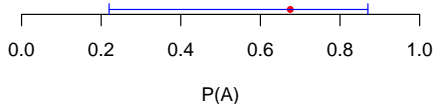
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generalizations:

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interval probability



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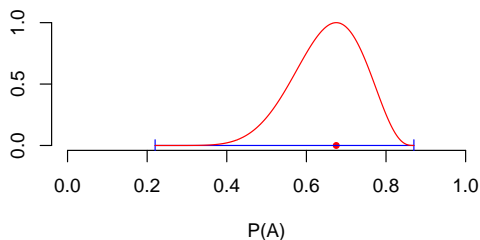
precise probability



interval probability



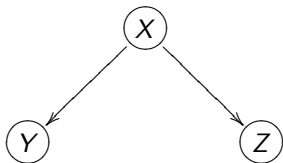
fuzzy probability



probabilistic graphical models

$X, Y, Z \in \{0, 1\}$

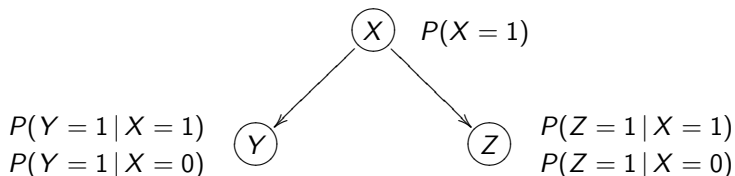
Y and Z independent conditional on X :



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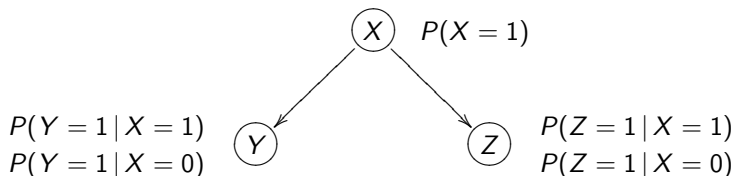
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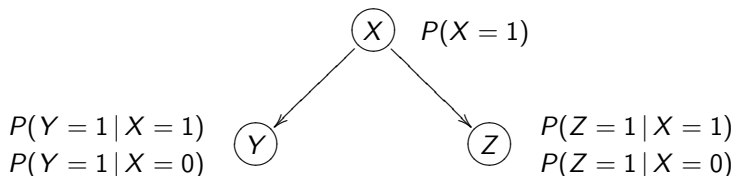
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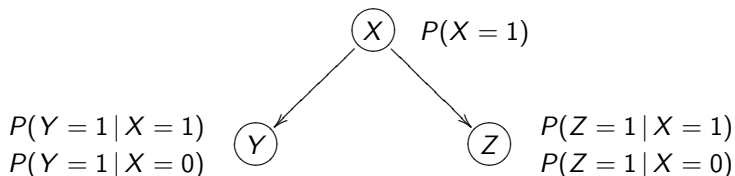


interval probabilities: credal networks

probabilistic graphical models

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generalizations:

precise probabilities: Bayesian networks



interval probabilities: credal networks



fuzzy probabilities: hierarchical networks

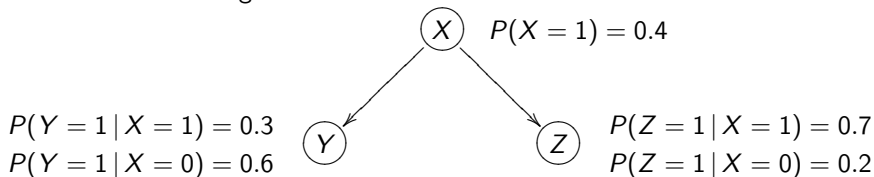
training data

X	Y	Z	#
0	0	0	21
0	0	1	6
0	1	0	30
0	1	1	7
1	0	0	9
1	0	1	15
1	1	0	5
1	1	1	7
			100

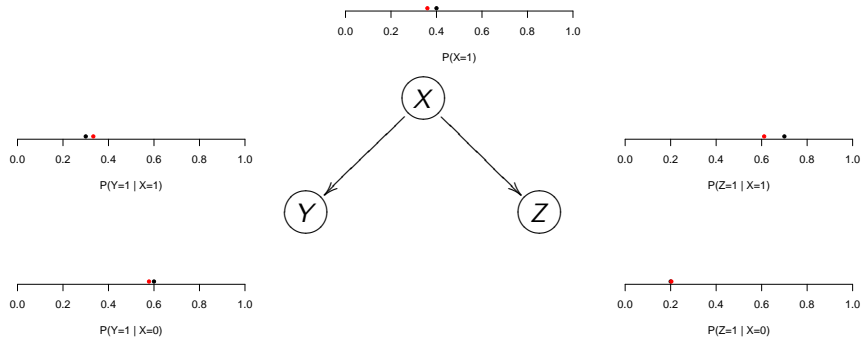
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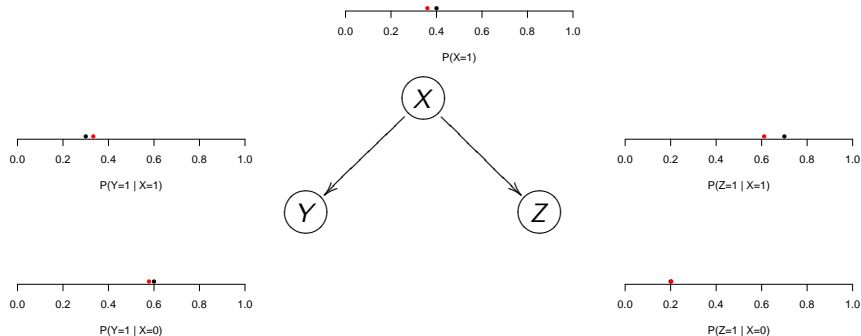
simulated according to:



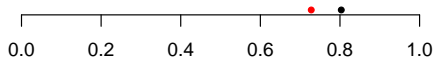
Bayesian network via MLE



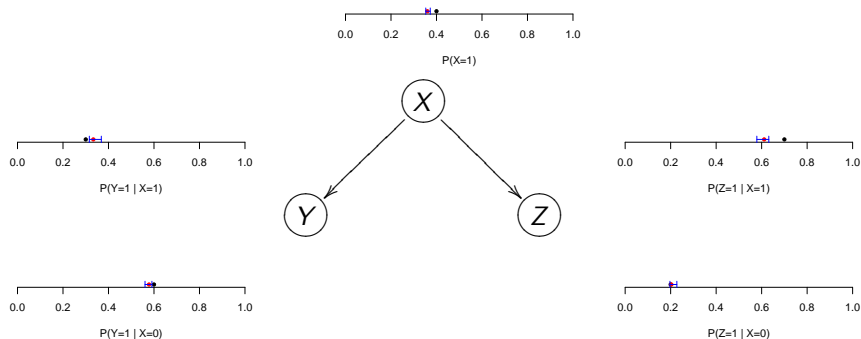
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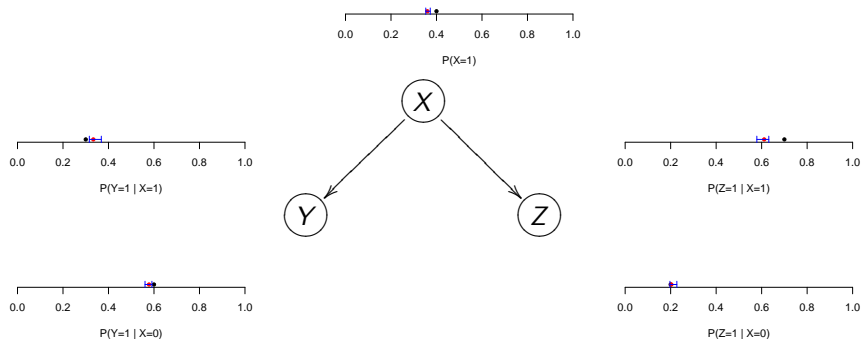
$\Rightarrow P(X = 1 | Y = 0, Z = 1) :$



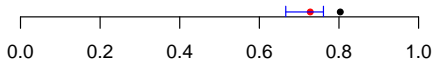
credal network via IDM (with $s = 2$)



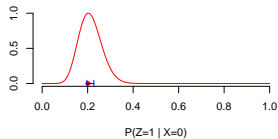
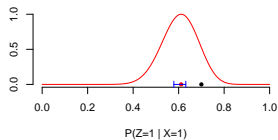
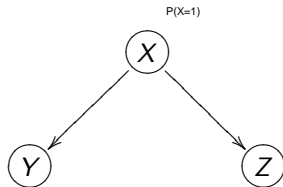
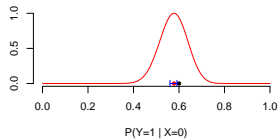
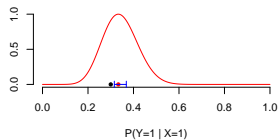
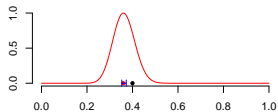
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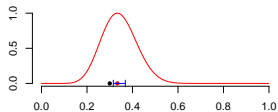
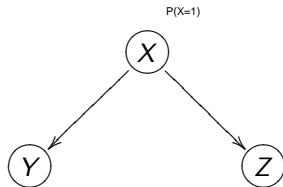
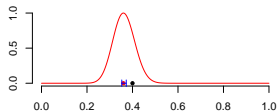
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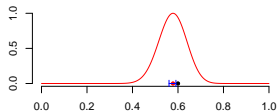
hierarchical network



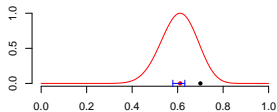
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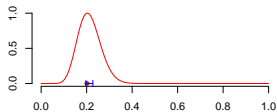
$P(Y=1 | X=1)$



$P(Y=1 | X=0)$

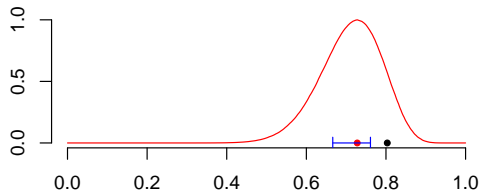


$P(Z=1 | X=1)$

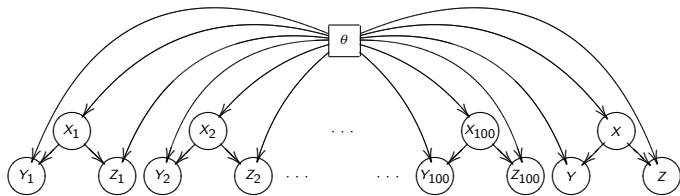


$P(Z=1 | X=0)$

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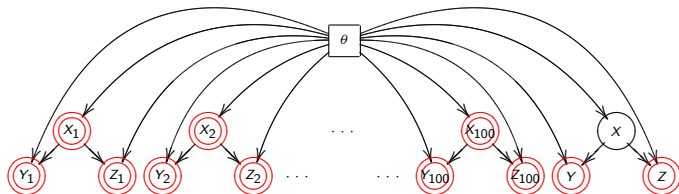


global models



prior ignorance
about $\theta \in [0, 1]^5$

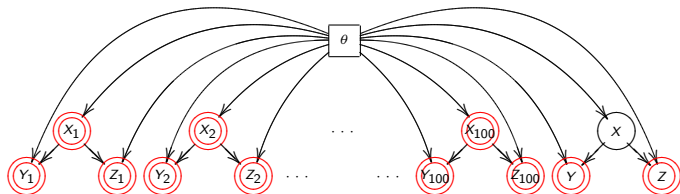
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302 variables
observed

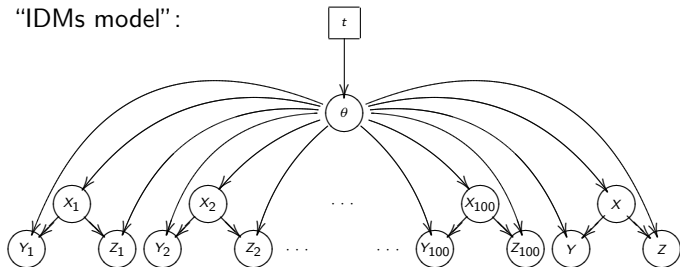
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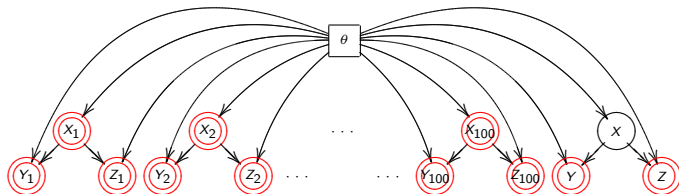
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“IDMs model”:



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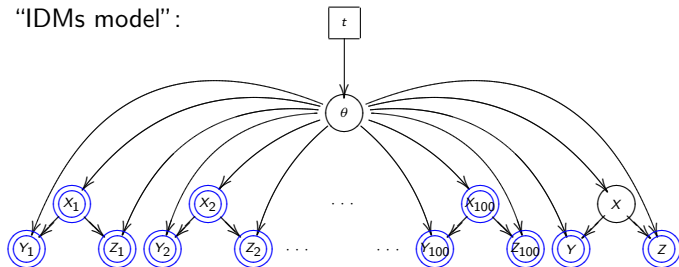
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conclusions and results

advantages of (likelihood-based) **fuzzy probability** over interval probability:

- ▶ **more expressive** (relative plausibility of different values in the probability interval)
- ▶ **more powerful updating rule** (extracts more information from the data)
- ▶ **more robust updating rule** (less sensitive to small perturbations of the model)

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mathematical results of the paper:

- ▶ **d-separation** implies conditional irrelevance in hierarchical networks
- ▶ hierarchical networks can be described by **convex sets of measures**, and it suffices to consider the extreme points