Marco Cattaneo

Combining Belief Functions Issued from Dependent Sources

The information issued from a source is described by a (normalized) belief function.

In order to pool the information issued from two sources, we have to combine the respective belief functions.

If we assume the independence of the sources, we can use Dempster's rule of combination.

(Notice that the independence assumption can be justified only by analogies with other situations in which it proved to be sensible.)

What if we assume nothing about the sources?

In this case, it is better to play safe and consider a conservative combination rule.

bba: $\quad m : \mathcal{P}^*(\Omega) \to [0,1]$ s.t. $\sum_A m(A) = 1$

possible combinations of m_1 and m_2:

jba: $\quad \underline{m} : \mathcal{P}^*(\Omega) \times \mathcal{P}^*(\Omega) \to [0,1]$ s.t.

$$\sum_B \underline{m}(A,B) = m_1(A), \ \sum_A \underline{m}(A,B) = m_2(B)$$

	$m_2(A)$	$m_2(B)$	$\cdots$
$m_1(A)$			
$m_1(B)$			
$\vdots$			

conflict: $\quad c(\underline{m}) = \sum_{A \cap B = \emptyset} \underline{m}(A,B)$

$$\leadsto \quad m(A) = \frac{1}{1 - c(\underline{m})} \sum_{B \cap C = A} \underline{m}(B,C)$$

The conflict is a good index for the non-monotonicity of the combination (in particular, no conflict implies monotonicity).

minimal conflict: $\quad \max_{A \subseteq \Omega} \Big(bel_1(A) - pl_2(A) \Big)$

A conservative combination rule should maintain as much as possible of both belief functions, without adding anything unnecessary.

$\rightsquigarrow$ minimize the conflict and then minimize the specificity

This is a problem of linear programming, but the solution is not always unique: the solutions build a convex polytope.

$\rightsquigarrow$ choose for instance its centre

The obtained rule $\odot$ satisfies:

- commutativity: $bel_1 \odot bel_2 = bel_2 \odot bel_1$
- monotonicity (if possible): $bel_1 \odot bel_2 \geq bel_1$
- $bel_1 \odot bel_2$ is a least specific common specialization of bel_1 and bel_2 (if possible)
  - absorption: bel_s spec. $bel \Rightarrow bel_s \odot bel = bel_s$
  - idempotency: $bel \odot bel = bel$

But minimization of conflict and idempotency are both incompatible with associativity.

Thus the binary rule $\odot$ is not associative, but it can be easily extended to an n-ary rule for the simultaneous combination of any number of belief functions.

In the generalized Bayes' theorem, the conservative combination rule $\odot$ leads to better results than Dempster's one.

Consider n hypotheses $h_1,...,h_n$ implying the belief functions $bel_1,...,bel_n$ on Ω, respectively.

Let the belief function bel_o on Ω represent an observation and let $c_1,...,c_n$ be the conflicts of its combination with $bel_1,...,bel_n$, respectively.

In the simplest case, the prior belief function on $\{h_1,...,h_n\}$ is an epistemic probability $p_1,...,p_n$. In this case, the posterior belief function is the epistemic probability $p'_1,...,p'_n$, with

$$p'_i \propto (1 - c_i)\, p_i.$$

Thus the conflicts come out as the measure of the disagreement between the respective hypotheses and the observation.

If the c_i are the minimal conflicts, then from $bel_o \leq pl_i$ follows $p'_i \geq p_i$.

This is not assured if we use Dempster's rule: $p'_i < p_i$ is possible even if $bel_o = bel_i$.