The Ways of Chance

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Mathematics Masterclass
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games of chance

$$\text{probability of an event} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

e.g., probability of rolling an even number with a die

$$= \frac{\text{⚂⚃⚅}}{\text{⚀⚁⚂⚃⚄⚅}} = \frac{3}{6} = \frac{1}{2} = 0.5 = \frac{50}{100} = 50\,\%$$

↕ law of large numbers

$$\text{probability of an event} \approx \frac{\text{number of times the event occurred}}{\text{number of trials}}$$

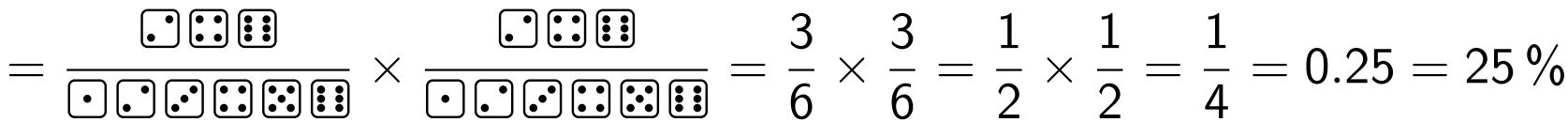
e.g., probability of rolling an even number with a die

$$\approx \frac{\text{number of times the outcome is an even number}}{\text{number of times the die is rolled}}$$

multiplication rule

probability of two independent events = probability of the first event
× probability of the second event

e.g., probability of rolling two even numbers with two dice



national lottery

probability of choosing the right 6 numbers out of 49?

national lottery

probability of choosing the right 6 numbers out of 49?

$$\frac{6}{49} \times \frac{5}{48} \times \frac{4}{47} \times \frac{3}{46} \times \frac{2}{45} \times \frac{1}{44} = \frac{720}{10068347520} = \frac{1}{13983816}$$

birthday paradox

probability that in a group of 30 people someone else has the same birthday as me?

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$1-$ (probability that in a group of 30 people no one has the same birthday as me)

$$= 1 - \underbrace{\frac{364}{365} \times \frac{364}{365} \times \frac{364}{365} \times \cdots \times \frac{364}{365}}_{29 \text{ times}} = 1 - \left(\frac{364}{365}\right)^{29} \approx 1 - 0.924 = 0.076 = 7.6\,\%$$

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probability that in a group of 30 people at least two have the same birthday?

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probability that in a group of 30 people at least two have the same birthday?

$1-$ (probability that in a group of 30 people everyone has a different birthday)

$$= 1 - \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \cdots \times \frac{365 - 29}{365} \approx 1 - 0.294 = 0.706 = 70.6\,\%$$

birthday paradox

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BBC: The birthday paradox at the World Cup

problems

what is the probability that:

- in your group someone else was born in the same month as you?
- in your group at least two people were born in the same month?
- at least one of your 6 numbers is right in the national lottery?

problems

what is the probability that:

- in your group someone else was born in the same month as you?

  e.g., group of 5 people: $1 - \left(\frac{11}{12}\right)^4 \approx 1 - 0.706 = 0.294 = 29.4\,\%$

- in your group at least two people were born in the same month?

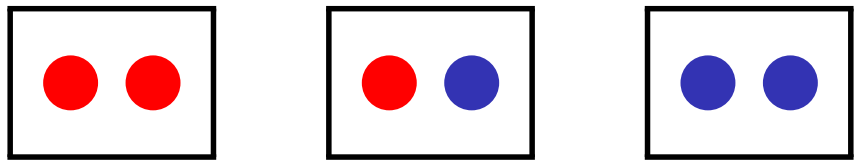
  e.g., group of 5 people: $1 - \frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} \approx 1 - 0.382 = 0.618 = 61.8\,\%$

- at least one of your 6 numbers is right in the national lottery?

  $1-$ (probability that no number is right)

  $= 1 - \frac{43}{49} \times \frac{42}{48} \times \frac{41}{47} \times \frac{40}{46} \times \frac{39}{45} \times \frac{38}{44} = \frac{563383}{998844} \approx 0.564 = 56.4\,\%$

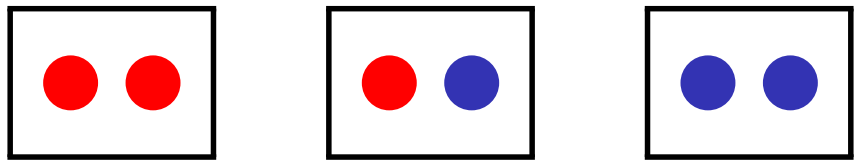
Bertrand's box paradox



choose a box at random and take one marble at random from the box

e.g., it is red: what is the probability that the remaining marble is also red?

Bertrand's box paradox



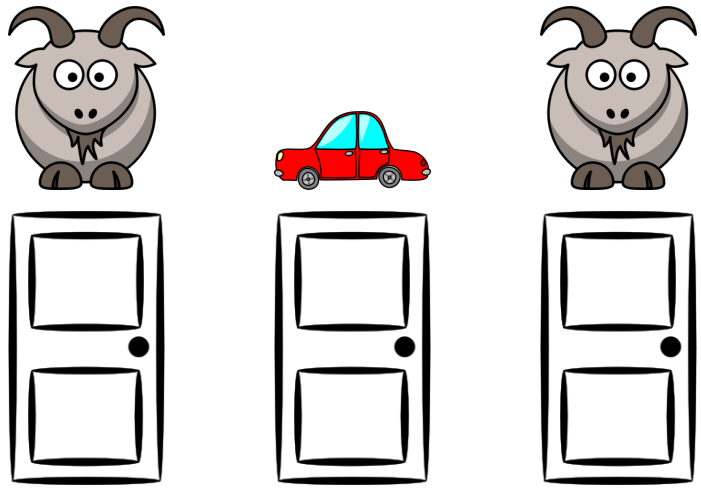
choose a box at random and take one marble at random from the box

e.g., it is red: what is the probability that the remaining marble is also red?

probability that the remaining marble has the same colour

$=$ probability of choosing a box with two marbles of the same colour $= \frac{2}{3}$

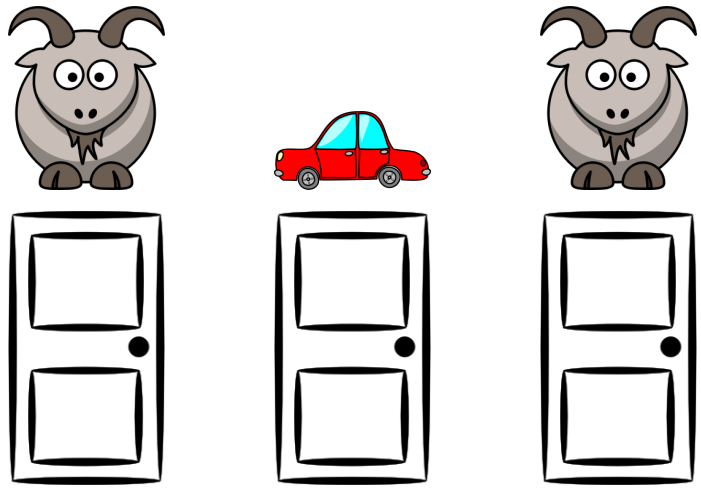
Monty Hall paradox



choose a door and the host will open one of the other doors to reveal a goat

e.g., you choose the first and he opens the third: should you switch to the second?

Monty Hall paradox



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probability of winning by switching doors

$= \text{probability that your first choice was wrong} = \frac{2}{3}$

problems

I choose two marbles at random and I will win if they are the red and the blue ones

what is the probability that I will win if:

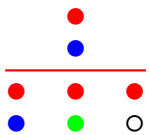
- I tell you that I have (at least) the red marble?
- I tell you that I have (at least) the blue marble?
- I tell you that I have (at least) one of them?

problems

I choose two marbles at random and I will win if they are the red and the blue ones

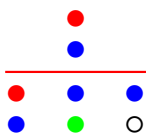
what is the probability that I will win if:

- I tell you that I have (at least) the red marble?



$= \frac{1}{3}$

- I tell you that I have (at least) the blue marble?



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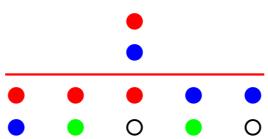
- I tell you that I have (at least) one of them?



$= \frac{1}{5}$