Likelihood theory as a unified approach to uncertainty

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  - **likelihood**: uncertain knowledge about θ described by λ, without need of prior information ("prior-free Bayesian" approach)

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  - **likelihood approach**: decisions and inferences obtained by minimizing the integral of the loss (or error) with respect to the possibility measure Λ [Cattaneo, 2013a]
- the likelihood function gives an interpretation to possibility measures and fuzzy sets [Cattaneo, 2008]

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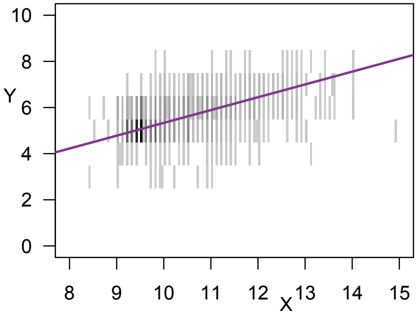
- **example**: relationship between the (subjective) sensory quality Y and the alcohol content X (in percent by volume) of $n = 1599$ Vinho Verde red wines from Portugal



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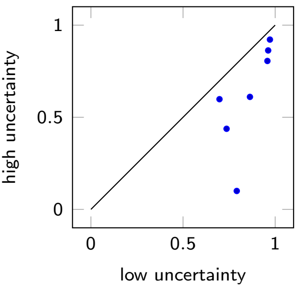
- these independences and the corresponding (conditional) probabilities must be **learned from data**
- the likelihood approach can describe the uncertainty about the probability estimates [Cattaneo, 2010]
- **example**: classifier in which a class is returned only when the probabilities can be estimated with sufficient certainty [Antonucci, Cattaneo, and Corani, 2012]

accuracy of the classification:



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- probabilistic graphical models:
  - simultaneous learning of independences and probabilities
  - application to expert systems

references

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