# Likelihood Decision Functions

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- statistical model: $(\Omega, \mathcal{F}, P_\theta)$ with $\theta \in \Theta$ (where $\Theta$ is a nonempty set) and random variables $X : \Omega \to \mathcal{X}$ and $X_i : \Omega \to \mathcal{X}_i$

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- these methods do not fit well in the setting of statistical decision theory: here they are unified (and generalized) in likelihood decision theory

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- strong similarity with the Bayesian approach (both satisfy the likelihood principle): a fundamental advantage of the likelihood approach is the possibility of not using prior information (since $\lambda_{x_1} \equiv 1$ describes complete ignorance)

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- likelihood decision function: $\delta : \mathcal{X} \to \mathcal{D}$ such that $\delta(x)$ minimizes $V(W(\cdot, d), \lambda_x)$

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  - asymptotic optimality: under some regularity conditions, the likelihood decision functions $\delta_n : \mathcal{X}_1 \times \cdots \times \mathcal{X}_n \to \mathcal{D}$ satisfy

$$\lim_{n \to \infty} W(\theta, \delta_n(X_1, \ldots, X_n)) = \inf_{d \in \mathcal{D}} W(\theta, d) \quad P_\theta\text{-a.s.}$$

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  - the likelihood ratio test with critical value $c'/c$ is the likelihood decision function resulting from the MPL criterion

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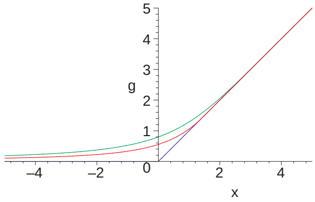

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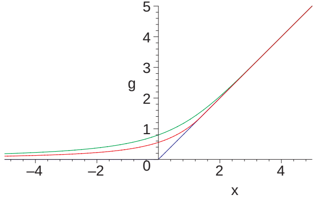


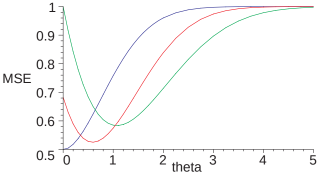

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  - does not need prior information