Linear regression with interval data: the LIR approach

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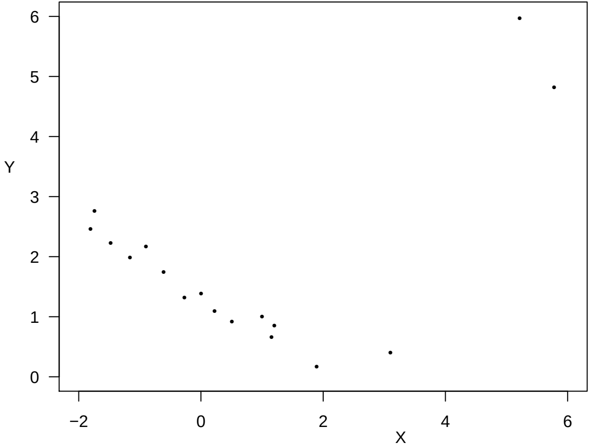

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- simple linear regression:

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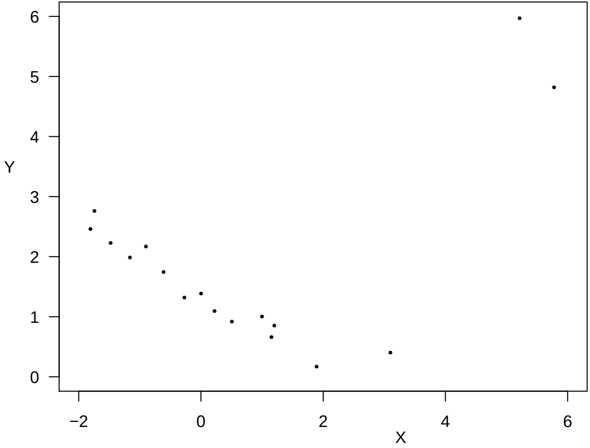

(Simple) linear LIR with interval data

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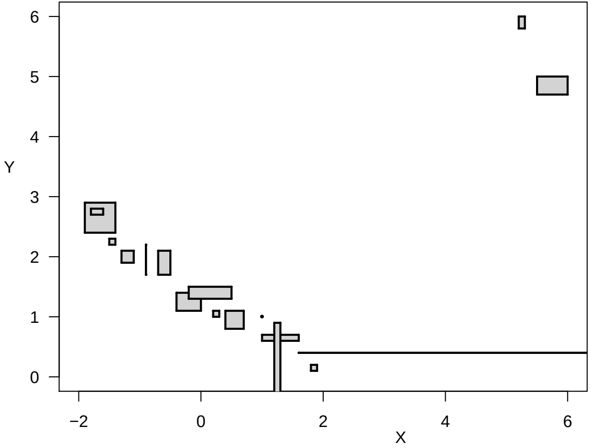

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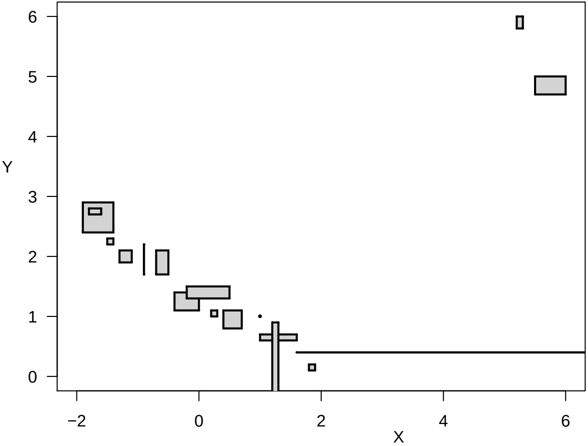

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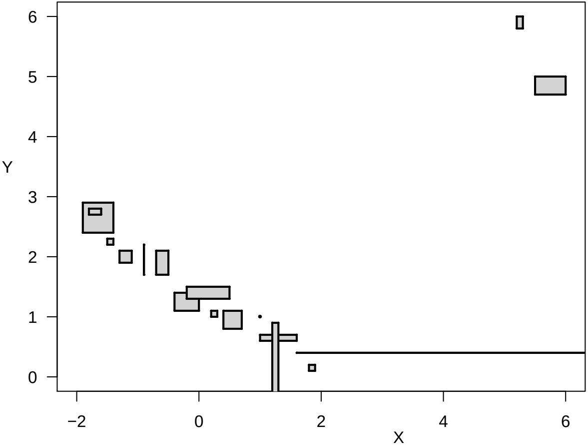

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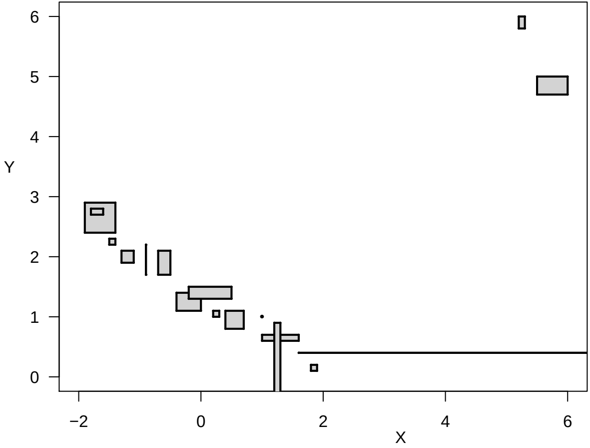
  $R_{f,i} = |Y_i - f(X_i)|$



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- imprecise residuals:

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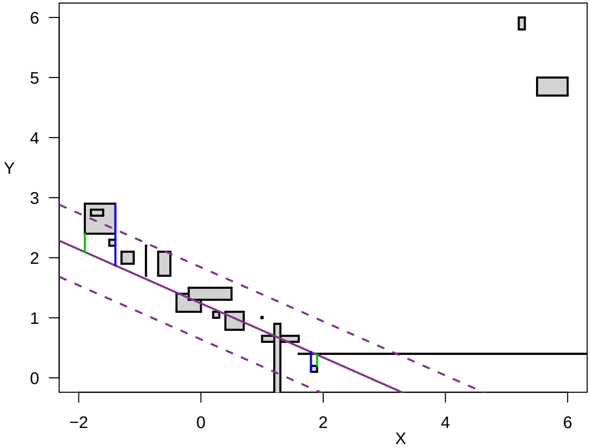

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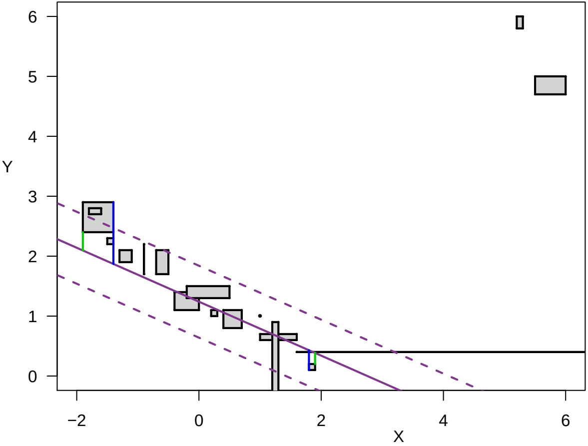

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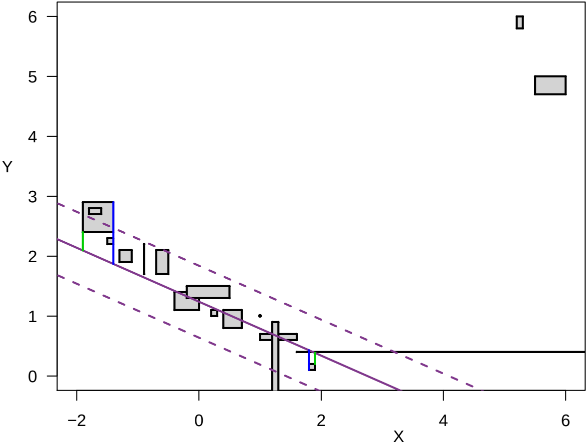

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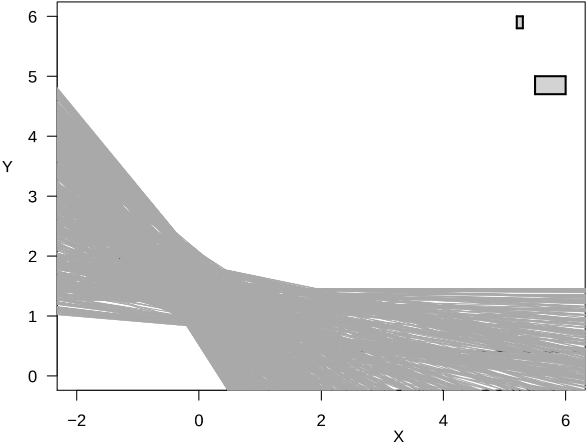
- result $\mathcal{U}$: set of all plausible functions



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- further details in: M. Cattaneo, A. Wiencierz (2012). *Likelihood-based Imprecise Regression*. Int. J. Approx. Reasoning 53. 1137-1154.

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- exact confidence level of $\mathcal{C}_f$:

$$\inf_{P \in \mathcal{P}_\varepsilon} P(\mathcal{C}_f \ni Q_{R_f}) = \begin{cases} \sum_{k=\underline{k}+1}^{\overline{k}} \binom{n}{k} p^k (1-p)^{n-k} & \varepsilon = 0 \\ \sum_{k=\underline{k}+1}^{\overline{k}} \binom{n}{k} (p+\varepsilon)^k (1-(p+\varepsilon))^{n-k} & \varepsilon > 0, p \leq 0.5 \\ \sum_{k=\underline{k}+1}^{\overline{k}} \binom{n}{k} (p-\varepsilon)^k (1-(p-\varepsilon))^{n-k} & \varepsilon > 0, p > 0.5 \end{cases}$$

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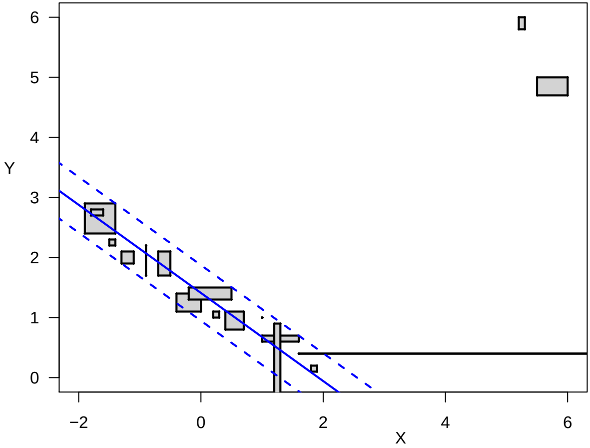
- 1st step: find $\overline{q}_{LRM}$
- $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$ (blue dashed lines) is the thinnest band containing at least $\overline{k}$ imprecise data
- here $\beta = 0.8, p = 0.6$, $n = 17$, and $\overline{k} = 12$



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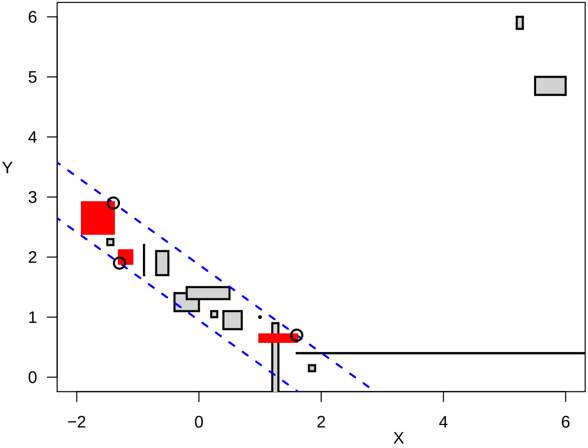
- some of the included $\overline{k}$ imprecise data touch the border of $\overline{B}_{f_{LRM},\overline{q}_{LRM}}$ in 3 different points



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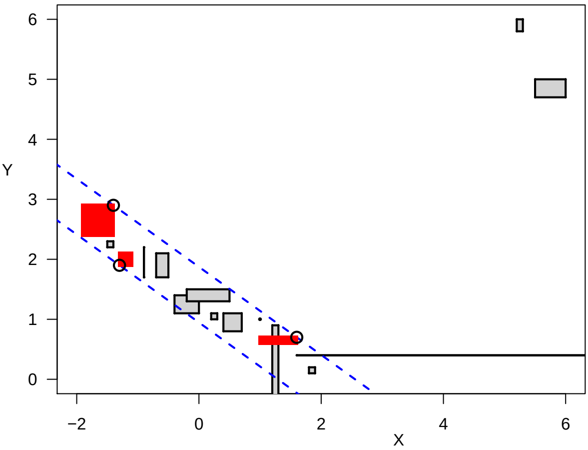

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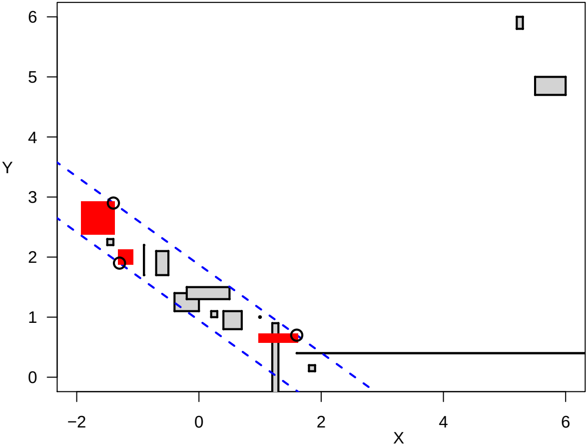

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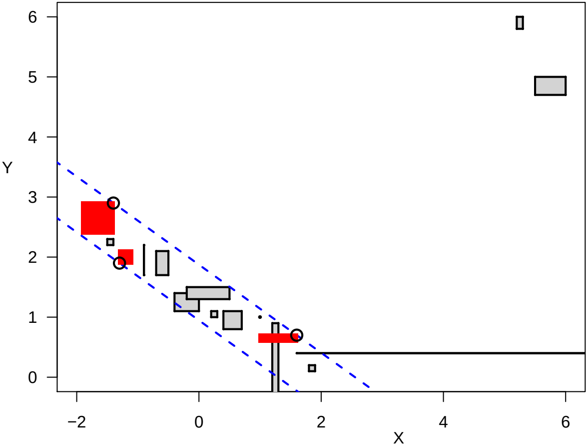

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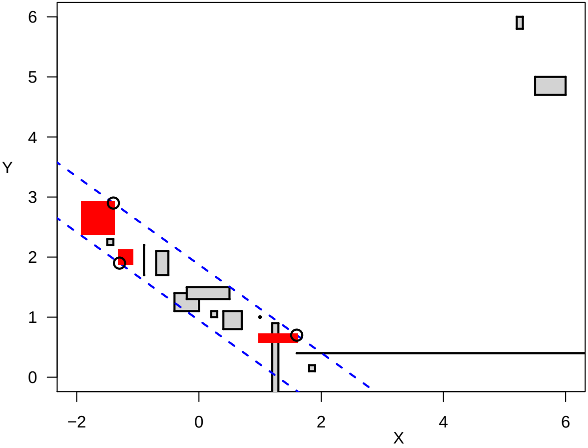

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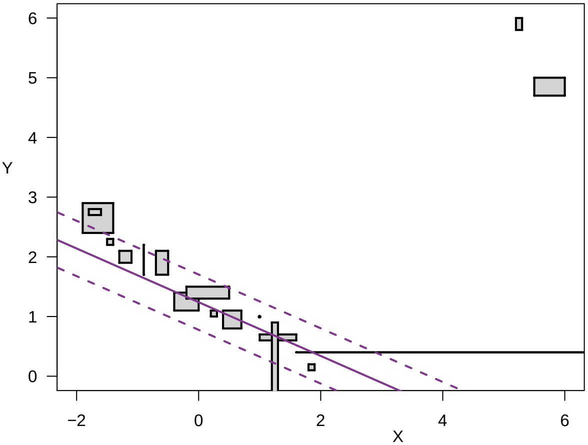

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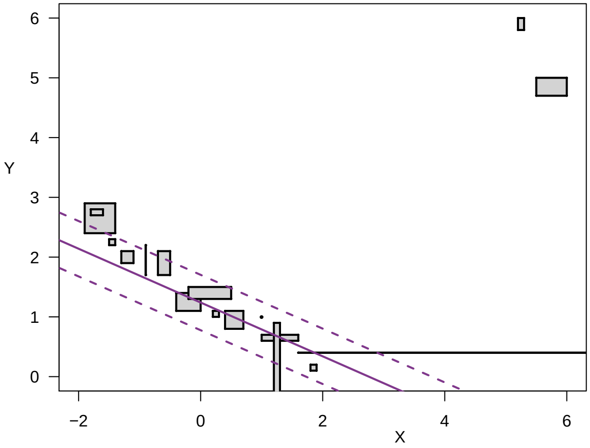
- step 2: determine $\mathcal{U}$
- if $f \in \mathcal{U}$, then $\overline{B}_{f,\overline{q}_{LRM}}$ intersects at least $\underline{k}+1$ imprecise data
- here $\underline{k} = 8$



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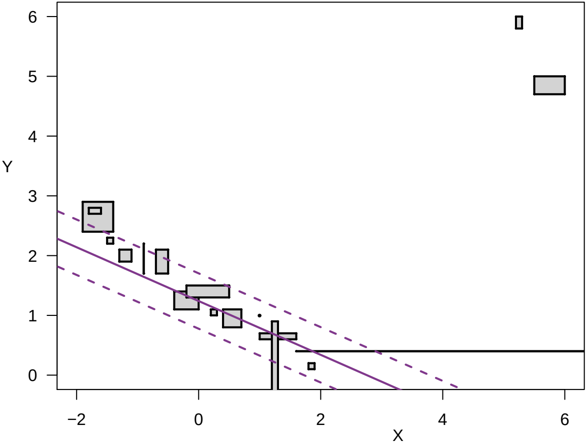
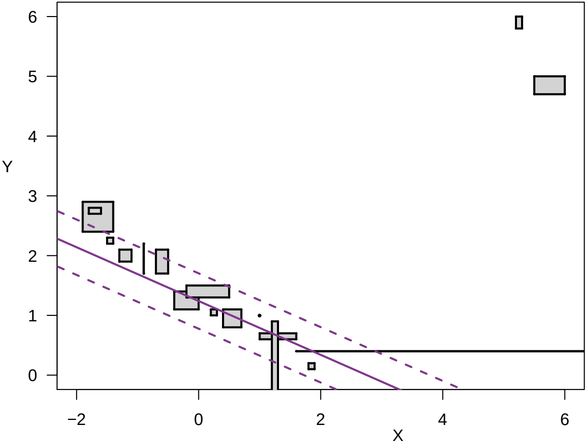
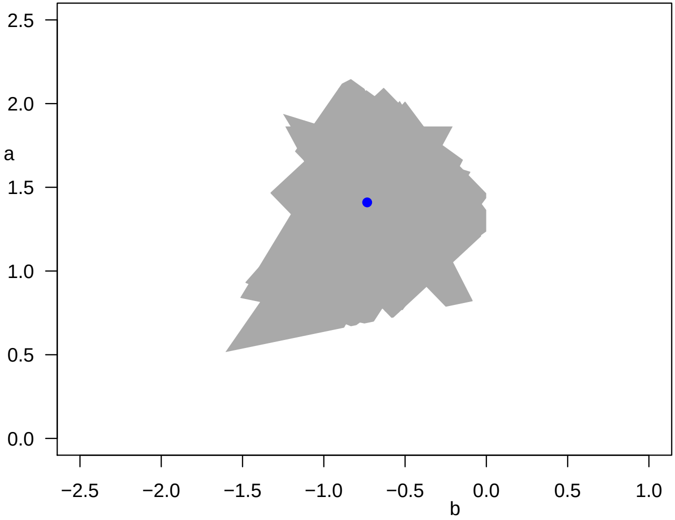


Implementation: Exact algorithm - Part 2

- step 2: determine $\mathcal{U}$
- if $f \in \mathcal{U}$, then $\overline{B}_{f,\overline{q}_{LRM}}$ intersects at least $\underline{k}+1$ imprecise data
- here $\underline{k} = 8$
- for each $b \in \mathbb{R}$ find the of intercept values $a \in \mathbb{R}$, for which $\underline{r}_{f_{a,b},(\underline{k}+1)} \leq \overline{q}_{LRM}$
- $\mathcal{U}$ can also be represented by the corresponding subset of the parameter space



Set of undominated parameters



Implementation of the algorithm in R

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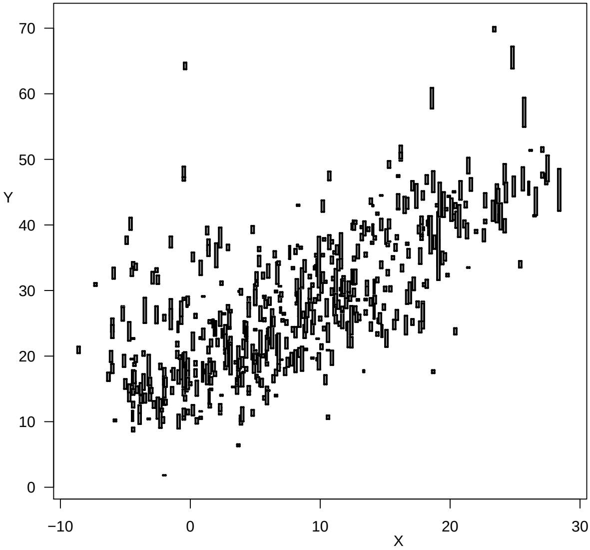
- further tools to summarize and visualize results

Example

- 2-dimensional interval data set of $n = 514$ observations



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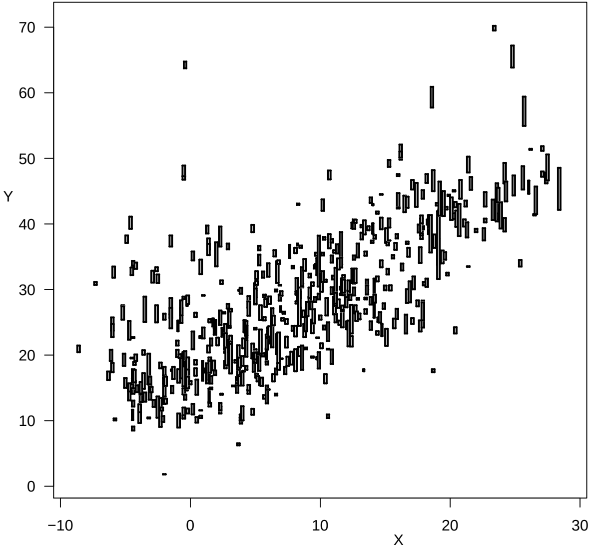

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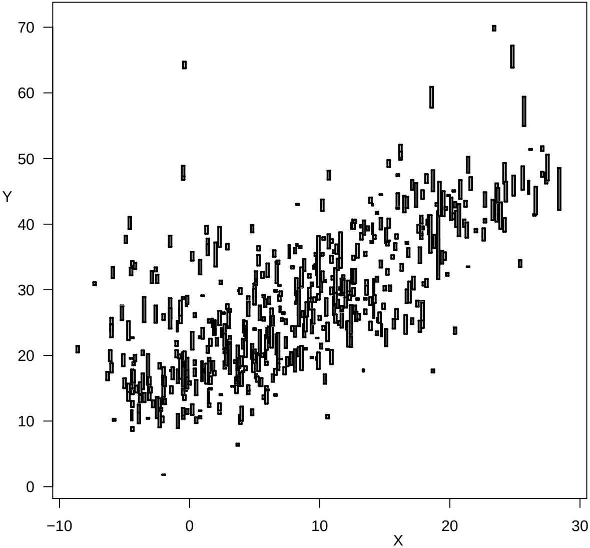

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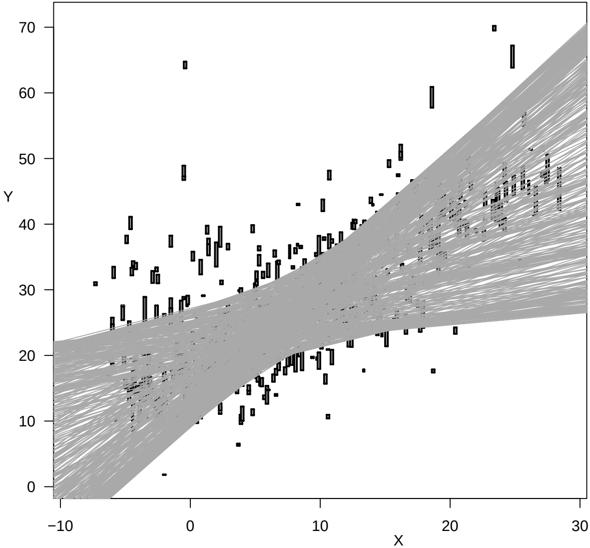

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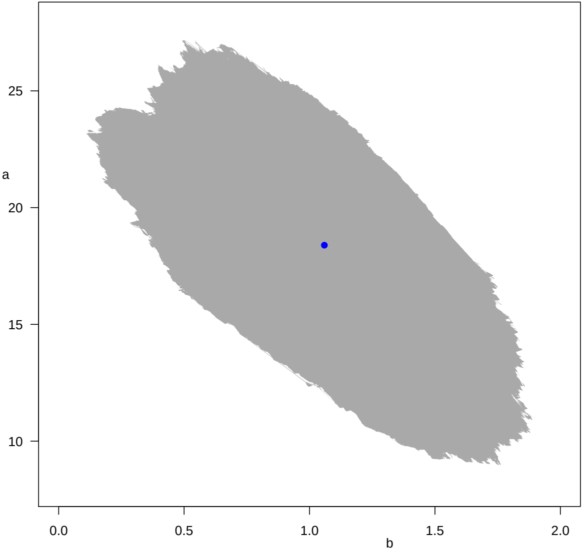

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